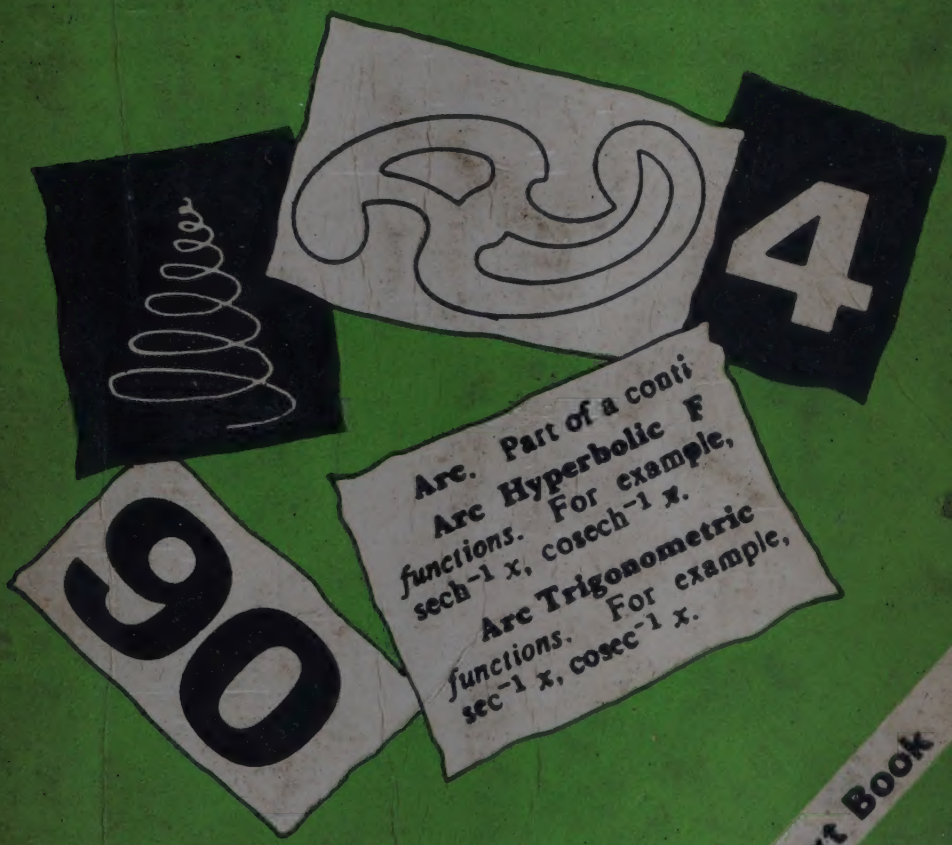
Dictionary of MATHEMATICS

(WITH MATHEMATICAL TABLES)



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Low Priced Text Book

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OF
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PREFACE

This *Dictionary* caters to the needs of the students pursuing the study of *Science* and *Mathematics* at School and College levels. The present volume will also prove very useful to those interested in acquiring efficiency in Mathematics. The dictionary contains about 2,000 headwords, used very commonly by students studying various courses of Mathematics. The important terms and mathematical principles are fully illustrated.

An attempt has been made to include most of the words and concepts, used upto degree level in mathematics. The author would be highly pleased to receive comments, about the omissions of serious nature, the contents of the enteries or any misprints.

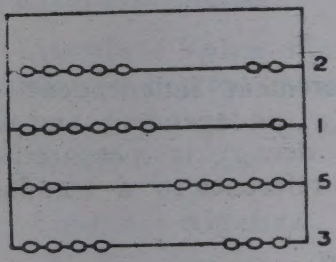
It is pleasure to thank friends and colleagues, for their help encouragement, cooperation and suggestions.

I am also very much thankful to the Pubishers and the Printers, who made great efforts to bring out the book in the present form.

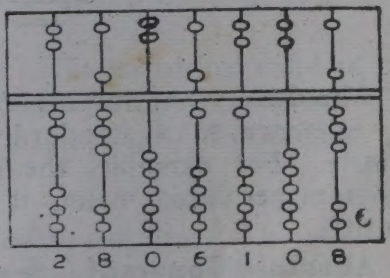
—AUTHOR

A

Abacus. The earliest and the simplest calculating device. Itsfirst version consisted of a set of grooves in the sand in which pebbles were used to represent units, tens, hundreds, according to their position in the groove. This device was extensively used by the Egyptians, Greeks, and Romans. The use of Abacus later spread throughout the civilized world in modified forms. The Japanese Abacus consisted of rows of beads strung on wires and mounted in a frame. The Abacus used in China and Bulgaria is divided into two parts with sets of two and five beads. The upper part was called *heaven* and the lower part *earth*. The beads in the upper part in the upper position denote 0 and those in the lower position 5. The beads in the lower part denote the unit digit.



Abacus showing the number 2153



Abacus showing the number 2806108

The Japanese Abacus operators are known to have won races against desk calculator operators.

Abberation. The change in the position of a heavenly body due to observer's velocity.

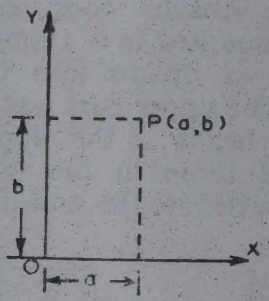
Abelian Group. See *Group* (commutative).

Abel's Theorem. See *Convergence.*

Abinitio. *Syn. From first principles* (by definition). Usually used in calculus. Used in establishing a result or formula from definition. It is particularly used in obtaining derivatives or integrals. For example, differentiate x^n abinitio.

Above Par. See *Premium.*

Abscissa. The x-coordinate (or horizontal coordinate) of a point in a plane. Thus, in the figure below the distance a represents the abscissa of the point P w.r.t. the origin O. See *Origin.*



Absolute. A number or measurement independent of any reference to a standard value, *i.e*, independent and not relative. For example, the absolute density is measured in kilogram per cubic meter, it has no reference to a standard density.

Absolute Constant. See *Constant.*

Absolute Convergence. The series $\sum_{n=1}^{\infty} u_n$ has absolute convergence if $\sum_{n=1}^{\infty} |u_n|$ converges. For example, the series $1-\frac{1}{2}+\frac{1}{4}-\frac{1}{8}+\cdots$ has absolute convergence because the

series $1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\cdots$ is convergent being a geometric series with common ratio less than 1. See *Convergence.*

Absolute Error. The positive difference between the measured and the calculated value of a quantity. See *Error* and *Relative error.*

Absolute Maxima. See *Maxima.*

Absolute Minima. See *Minima.*

Absolute Number. Syn. *Absolute Constant.* A non-literal constant.

Absolute Term. Term (in an expression) free from unknown quantities. For example, in the equation

$$ax^2+by^2+2gx+2fy+c=0\ ;$$

c is the absolute term.

Absolute Value. Syn. *Numerical Value.* The value of a quantity without any reference to its sign or direction.

Absolute Value of a Compiex quantity. See *Modulus* of a complex number.

Absolute Value of a Vector. The numerical value of the length of a directed line segment representing the vector. The absolute value of the vector $ai+bj+ck$ is $\sqrt{a^2+b^2+c^2}$. See *Magnitude of a vector.*

Abstract Algebra. Syn. *Modern Algebra.*

Abstract Number. Syn. *Absolute Number.*

Abstract Space. A formalized mathematical system with geometrical properties.

Acceleration. Rate of change of velocity with time. A body which falls from rest in space, moves with an acceleration due to gravity, denoted by g which is 32.2 ft/sec^2 in FPS system and 980.665 cm/sec^2 in CGS system. The negative acceleration is also known as *retardation.* If an object moves

along a curved path with constant velocity even then it has an acceleration since velocity in that case changes its direction at every point.

In the case of motion of an object in a circular path the acceleration is $\frac{v^2}{r}$, where v is the velocity of the object and r the radius of the circle.

The uniform acceleration a in the linear motion of a body is given by

$$a=(v_2-v_1)/t$$

where v_1 is the initial velocity and v_2 the velocity after a time t.

Acceleration due to Gravity (g). The acceleration of a freely falling body due to the attraction of the earth. It is denoted by g. See Acceleration also.

Acceleration of Free Fall. See Acceleration due to gravity.

Acceleration Instantaneous. Suppose v is the velocity of an object at any time t and $v+\delta v$ at a time $t+\delta t$. The instantaneous acceleration of the object at a time t is the limiting value of $\frac{\delta v}{\delta t}$ as δt approaches 0. It is denoted by $\frac{dv}{dt}$ or $\frac{d^2x}{dt^2}$, where x is the distance covered by the object in time t.

Accountancy. The principles and adopted practices of maintaining accounts.

Accounts. Records of financial transactions.

Accumulation Points. Syn. *Cluster point, Limit point.* This point is the limit of a sequence of points in a given set. Thus, in set $\left\{ 1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots. \right\}$, the point 0 is an accumulation point.

Accuracy. This represents the number of significant digits while measuring or evaluating a quantity. For example, if a certain length is measured or evaluated as 13.5676 and the accuracy is limited to three places of decimal then the number taken will be 13.567.

Accurate to n places of decimal. See *Correct to n places of decimal.*

Acnode. Syn. *Isolated Point.*

Acre. British unit of area as 4840 sq. yards equivalent to 4046.8 sq. meters (.40468 hectare).

Action. Obselete term for force. When a force is exerted on a body, for example, by pushing, it is resisted by another force. Newton was the first to express the relationship between the force and the resisting force, called reaction. According to his third law of motion, every action has an equal and opposite reaction.

Actuary. A person who calculates insurance risks ; an expert in statistics.

Acute Angle. An angle less than a right angle (90° or $\pi/2$ radians).

Addend. Any one of the numbers to be added together to give a sum.

Addition. It is an arithmetic operation with symbol $+$. It is a binary operation which gives sum of two or more quantities.

Addition of Numbers. The addition of numbers is commutative $(a+b=b+a)$, associative $[a+(b+c)=(a+b)+c]$, the identity element is zero $(a+0=a=0+a)$. Inverse operation of addition is subtraction with symbol—defind as

$$a-b=a+(-b).$$

Addition of Matrices. Addition of two matrices is a binary operation. Two or more matrices can be added if they are of the same order. See *Matrix.*

Addition of Vectors. Addition of vectors is different conceptually since vectors have magnitude as well as direction. See *Vector*.

Addition Formulae. Relations that connect trigonometric functions of sum or difference of angles with individual functions of these angles.

$$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$$

$$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$$

$$\tan(x \pm y) = (\tan x \pm \tan y)/(1 \mp \tan x \tan y)$$

From these relations we can also have

$$\sin 2x = 2 \sin x \cos x$$

$$\cos 2x = \cos^2 x - \sin^2 x = 1 - 2 \sin^2 x = 2 \cos^2 x - 1$$

$$\tan 2x = 2 \tan x/(1 - \tan^2 x)$$

$$\sin 3x = 3 \sin x - 4 \sin^3 x$$

$$\cos 3x = 4 \cos^3 x - 3 \cos x$$

$$\tan 3x = (3 \tan x - \tan^3 x)/(1 - 3 \tan^2 x)$$

$$2 \sin x \cos y = \sin(x+y) + \sin(x-y)$$

$$2 \cos x \sin y = \sin(x+y) - \sin(x-y)$$

$$2 \cos x \cos y = \cos(x+y) + \cos(x-y)$$

$$2 \sin x \sin y = \cos(x-y) - \cos(x+y)$$

Addition of Complex numbers. See *Complex Numbers*.

Additive Function. Any function f for which we could write $f(x+y) = f(x) + f(y)$.

Ad Infinitum. Counting of terms under addition upto infinity. Usual notation is $\ldots \infty$.

Adjacent. Two angles with common initial point and drawn by three rays in a diagram, are called *adjacent angles*. Also, any pair of sides in a polygon that meet at one vertex are called *adjacent sides*. See *Vertex*.

Adjoint. See *Matrix*.

Aerodynamics. See *Dynamics*.

Affine Geometry. Branch of projective geometry in which we study the properties of geometrical figures which remain unchanged by parallel (cylindrical) geometric projection.

Aggregate. A term used for totality or wholeness of a collection of certain objects. For example, aggregate of rational numbers. As in set theory the aggregate may be closed or ordered also.

Algebra. Branch of mathematics in which letters are used to denote numbers in arithmetic operations. For example, the relation, $x+y=y+x$ belongs to algebra since it is true for any two numbers denoted by x and y. This property is called commutativity. The associativity and distributive properties, described as

$$x+(y+z)=(x+y)+z$$

$$x(y+z)=xy+xz$$

also figure in *algebra*. In an equation any term can be added or substracted on both sides. Also, both sides of the equation may be multiplied or divided by any constant or variable quantity.

Algebra Boolean. See *Boolean Algebra*

Algebra of Propositions. The Boolean Algebra of logic which treats mathematically the validity of propositions involving the phrase connectives ; and ; or ; not ; if ; then ; if and only if ; then.

Conditional. If P then Q, written as $P \rightarrow Q$.

Conjunction. P and Q, written as $P \wedge Q$.

Denial (negation). Not P, written as P', $-P$, $\sim P$.

Disjunction. P or Q (or both), written as $P \vee Q$.

Equivalence or Biconditional. If P and only P, then Q written as iff P then Q, $P \leftrightarrow Q$, $P \equiv Q$, or $P \sim Q$.

The rules of operation for algebra of propositions are isomorphic to those for algebra of sets, if the *union* is taken to be equivalent to the connective *or*, and the *intersection* to *and*.

Algebra of Sets. The Boolean Algebra of set theory. For a set of sets $\{A, B, C, \ldots\ldots\}$ subject to two binary opera-

tions, union ($\cup$) and intersection ($\cap$), with ϕ as the null set and I the universal set, the following operational rules apply.

Closure. $A \cup B$ is a set ; $A \cap B$ is a set.
Associative law. $A \cup (B \cup C) = (A \cup B) \cup C$,
$A \cap (B \cap C) = (A \cap B) \cap C$
Commutative law. $A \cup B = B \cup A$, $A \cap B = B \cap A$
Distributive law. $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
Idempotent. $A \cup A = A$, $A \cap A = A$
Bounds. $A \cup \phi = A$, $A \cup I = I$; $A \cap \phi = \phi$, $A \cap I = A$.
Complements. $A \cup A' = I$; $A \cap A' = \phi$; $(A \cup B)' = A' \cap B'$; $(A \cap B)' = A' \cup B'$. Example of De Morgan's law.
Involution. $(A')' = A$; $\phi' = I$; $I' = \phi$.

Algebraic Equation. An equation containing only algebraic expressions and signs of operations.

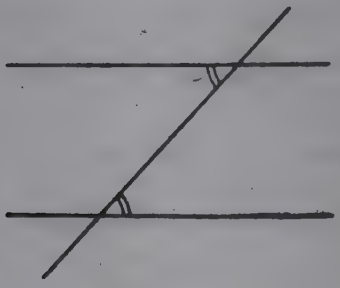
Algebraic Expression. Collection of variable names and constants which are connected by signs of operations. Those parts of the expression which are connected by plus or minus signs are called its terms.

Algebraic Function. A function involving only algebraic expressions and signs of operations.

ALGOL. A programming language for the computer.

Algorithm. Procedure for carrying out a desired computation in solving a problem, written sequentially in a series of steps.

Alternate Angles. A pair of equal angles formed by parallel lines and a third line crossing them. See *Transversal.*



Alternating series. An infinite series with terms alternately positive and negative. For example,

$$-1+\frac{1}{2}-\frac{1}{3}+\frac{1}{4}-\frac{1}{5}+\ldots\ldots$$

is an alternating series. An alternating series

$$u_1-u_2+u_3-u_4+\ldots\ldots$$

is convergent if from and after certain term each term is numerically less than its preceding term and $\underset{n\to\infty}{\text{Lt}}\ u_n=0$.

Alternation. (1) In proportion, if $a/b=c/d$, the deduction that $a/c=b/d$. (2) In algebra of proportions; a synonym of *disjunction.*

Altitude. A line segment and its length drawn from a vertex perpendicular to the opposite side (called the base). If a rectilinear figure has n sides there will be $(n-2)$ altitudes from each vertex. If $n=3$, the figure will be a triangle and hence three altitudes will be there which will be concurrent.

Ambiguous. Having more than one possible meaning, value, or solution. For example, if we are given two sides and one angle other than the included angle of a triangle, it gives rise to an ambiguous case in finding the other side and angles of the triangle. One solution is an acute-angled triangle and the other is an obtuse-angled triangle.

Amount. (1) Quantity. (2) Principal plus interest. See *Simple interest* ; *Compound interest.*

Amplitude in Oscillatory motion. The maximum displacement of an oscillating body from its mean position.

Amplitude in Polar coordinates. See *Polar co-ordinates in Plane.*

Amplitude of Complex number. See *Polar form of complex numbers.*

Analog Computer. A type of computer in which data is represented by a physical quantity usually voltage which can vary continuously. The data taken from some problem may be a changing temperature or pressure in a system. The data in the

analog computer gets converted into its voltage analogue. The resulting voltage is proportional to the result of the operation, which is recorded.

Analog computers can perform much more complicated computations than digital computers but are not that accurate and are less flexible in their use.

Analogy. Similarity between any two things, more so in methods. It is used to obtain results of one problem from the known results of the other. For example, Napier's analogy

$$\frac{\tan \frac{1}{2}(B-C)}{\cot \frac{1}{2}A} = \frac{\sin \frac{1}{2}(B-C)}{\sin \frac{1}{2}(B+C)}.$$

Analysis. Branch of mathematics which is based on the theories of continuity, limits, convergence of series and of differentiation, integration, complex variables etc.

Analytic Function. A function $f(z)$ of a complex variable z is said to be analytic at a point $z=z_0$, if $f(z)$ is a single valued and $\frac{d}{dz} f(z)$ $\left(\text{defined as } \lim_{\delta z \to 0} \frac{f(z+\delta z)-f(z)}{\delta z} \right)$ exists and is unique whatever may be the path along which δz tends to zero.

Analytical Geometry. (Coordinate geometry). Branch of mathematics developed by Euclid, Appollonius, and Descartes. Points are defined by means of a frame of reference and coordinates. Cartesian coordinate system is represented by set of numbers. Thus, in the analytical or coordinate geometry, curves can be drawn from the given algebraic equation, and also, inversely, equations of the curves can be obtained as loci of points.

Angle (plane angle). The spatial relationship between two straight lines, *i.e.*, when a straight line rotates about a fixed point on it in a plane, the angle is the measure of rotation. As per the convention adopted, an anticlockwise rotation generates a positive angle while a clockwise rotation a negative angle.

Angle between two parallel lines is zero. Angles are measured in degrees, or alternatively, in radians.

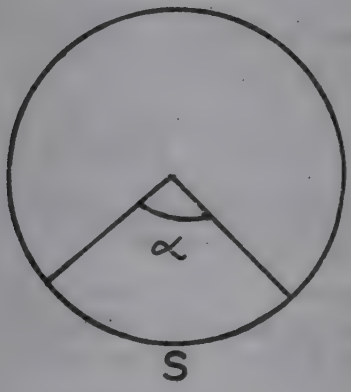
1 revolution = 360 degrees

1 degree = 60 minutes (first minute parts)

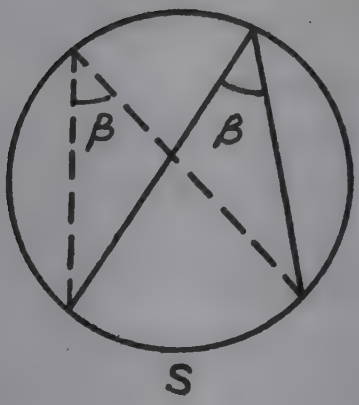
1 minute = 60 seconds (second minute parts).

A right angle is 90° while a straight line forms two right angles, *i.e.*, 180° or π radians.

Angle at Centre. The angle between any two radii of a circle. In the figure the arc *S* is said to subtend an angle α at the centre. The size of the angle at the centre is directly proportional to the length of the arc.



Angle at the Circumference. The angle between any two chords of a circle, having common point on the circumference of the circle. In the figure arc *S* is said to subtend an angle β at the circumference. For a given arc the angle at every point of the circumference will be the same. The angle at the circumference is half the angle at the centre for a given arc. See *Angle at the centre.*



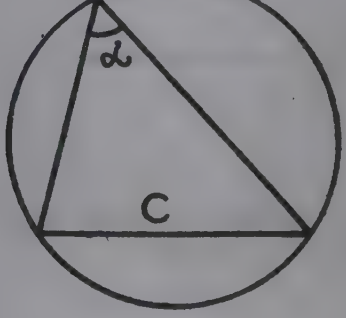
Angle between two Curves. The angle between the tangents to the curves at their points of intersection.

Angle between two Lines. Angle θ between two lines in a plane with slopes m_1 and m_2, is given by

$$\tan \theta = \frac{m_1 \sim m_2}{1 + m_1 m_2}.$$

Angle between two Planes. Angle between two planes is the angle between their normals.

Angle in Segment. Any interior angle of an inscribed triangle of a circle, the angle α subtended at the circumference by the chord. See *Angle at circumference.*



Angle in Semicircle. The angle in segment when the chord is a diameter. The angle in a semi-circle is always a right angle.

Angle of Depression. See *Angular levelling.*

Angle of Elevation. See *Angular levelling.*

Angle of Fire. The initial angle of inclination of a projectile.

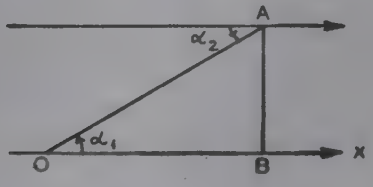
Angle of Friction. See *Friction.*

Angles Spherical. See *Spherical angles.*

Angular Acceleration. The rate of change of angular velocity, represented by $\frac{dw}{dt}$, or $\frac{d^2\theta}{dt^2}$, where ω is the angular velocity and θ is the angular displacement. See *Angular displacement.*

Angular Displacement. The angle of rotation measured from some initial direction. It is positive in anti-clockwise direction.

Angular Levelling. Determining differences in elevation of points on earth's surface. In the figure *OB* is horizontal and *AB* is vertical. Angle α_1 measures the *angle of elevation* of *A* from *O*. Angle α_2 measures the angle of depression of *O* from *A*.



Angular Motion. Syn. *Rotation.* See *Angular acceleration, Angular distance ; Angular velocity.*

Angular Velocity. The rate of change of angular distance denoted by ω or $\frac{d\theta}{dt}$.

Annular. Shaped like a ring.

Annulus. The portion of a plane bounded by the circumference of two concentric circles in the plane.

Annum. See *Per annum.*

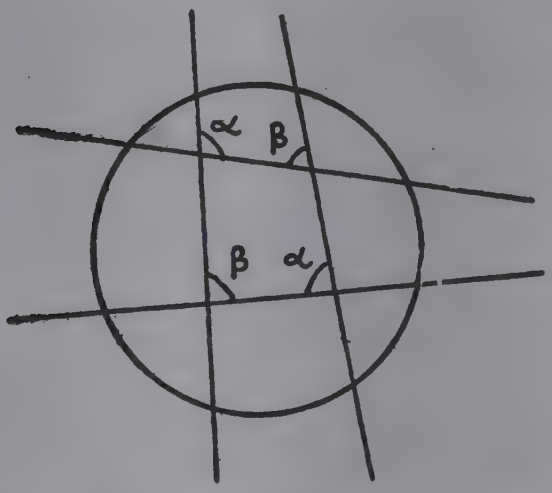
Answer. From the Anglo-saxon, to swear against, this word means to make a reply to a question or charge under oath.

Antecedent, Consequent in Logic. In the conditional preposition, if P then Q, P is the *antecedent* and Q the *consequent.*

Anti-Clockwise. See *Counter clockwise.*

Anti-Logarithms. (*Anti-log*). The inverse of logarithm. In common logarithm the anti-logarithm of x is 10^x. See *Logarithm.*

Anti-Parallel. Two lines L_1 and L_2 are said to be anti-parallel if they cut two given lines λ_1 and λ_2 to form two pairs of equal angles in opposite order.



Apex. The point at the top of a solid, or a plane figure with respect to a line or a plane chosen as base. For example,

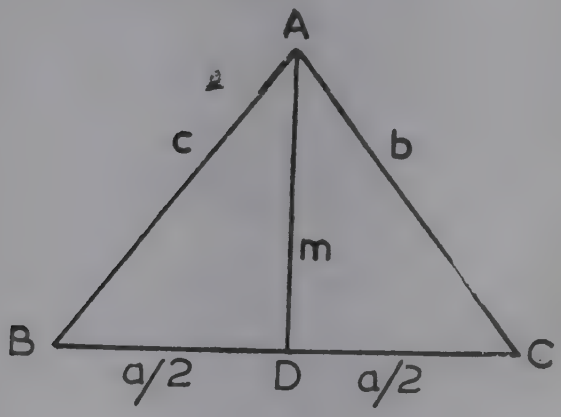
top of a solid like pyramid or the vertex of a triangle opposite the side chosen as base.

Apothem. A perpendicular from the centre of a regular polygon to mid-point of any side. In otherwords it is the radius of the inscribed circle of a regular polygon and is also called *short radius.*

Applied Mathematics. The study of the mathematical methods applied to solve physical problems. For example, studying about lines, points, angles, planes, etc. under certain axioms is pure geometry whereas the Euclidean geometry used in surveying, architecture, navigation, etc. is called applied geometry. The term 'applied mathematics' is used especially for mechanics.

Appolonius' Theorem. This theorem connects the length of a median in a triangle to the lengths of its sides. If AD is the median of the triangle ABC, of length m, then

$$b^2+c^2=2m^2+2\,(a/2)^2.$$



Approximation. A number which is taken as a close estimate of another number, or the method of finding such approximate number. The symbol used for approximation is $\approx$ or $\simeq$. For example, $\pi \simeq 3.1416$, $\sqrt{2} \approx 1.4142$.

Arbitrary Constant. See *Constant.*

Arrangements. Syn. *Permutations.*

Arc. Part of a continuous curve.

Arc Hyperbolic Function. Syn. *Inverse hyperboli functions.* For example, $\sinh^{-1} x$, $\cosh^{-1} x$, $\tanh^{-1} x$, $\coth^{-1} x$ $\operatorname{sech}^{-1} x$, $\operatorname{cosech}^{-1} x$.

Arc Trigonometric Functions. Syn. *Inverse trigonometri functions.* For example, $\sin^{-1} x$, $\cos^{-1} x$, $\tan^{-1} x$, $\cot^{-1} x$ $\sec^{-1} x$, $\operatorname{cosec}^{-1} x$.

Archimedes' Constant. See *Pi.* π.

Archimedes' Principle. Archimedes discovered tha when a body is partly or wholly immersed in a liquid, th body experiences an upward thrust, this thrust is a vertica force equal to the weight of the liquid displaced and it act through the centre of gravity of the space occupied originall by the displaced liquid.

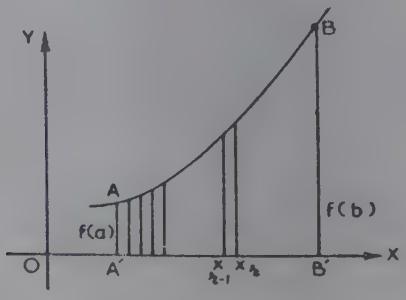
Archimedes Spiral. The curve whose polar equation $r=a\theta$.

Are. A larger unit of area in metric system ; equal 100 square meters.

Area. The size of a surface, that is, the extent of a plan figure or surface measured in units of $(\text{length})^2$. Area of rectangle is its length $\times$ breadth. The SI units of area sq. meter.

Area of Revolution. See *Solid of revolution.*

Area under the Curve. It is the area bounded by th curve $y=f(x)$, two ordinates $x=a$ and $x=b$, and the x-axis. is calculated by taking recourse to calculus. See *Calculus.*



The interval (a, b) is divided into n parts so that rth subinterval is $\delta x_r = x_r - x_{r-1}$. The area under the curve is approximated by

$\sum_{r=1}^{n} y_r \delta x_r \ (= \Sigma y \delta x)$ and is actually equal to $\lim_{n \to \infty} \sum_{r=1}^{n} y_r \delta x_r$

represented by $\int_a^b y\,dx \left(= \int_a^b f(x)\,dx \right)$.

Argand Diagram. See *Complex number*.

Argument. Syn. *Amplitude*. A complex number $z(=x+iy)$ can be written in polar form as $r(\cos\theta + i \sin\theta)$ where $r = +\sqrt{x^2+y^2}$, $\theta = \tan^{-1} y/x$, then θ is called the argument of z. It is, therefore, the angle that the vector representing the complex number, makes with the x-axis in the Argand diagram. See Complex number.

Arithmetic. Branch of mathematics which deals with numerical aspects of numbers and which involves the study of the skills necessary to manipulate numbers. In arithmetic we understand the structure of number system. It facilitates to change the numbers from one form to another. For example, the numbers in the form of fraction can be changed to decimal forms and vice versa.

Arithmetic Average. See *Arithmetic mean*.

Arithmetic and Logic Unit. Syn. *ALU*. See *CPU*.

Arithmetic Mean(s). The terms of an arithmetic progression, lying betweeen the first and the last are called arithmetic means of the first and the last terms. To insert n means between two numbers a and b implies having $n+2$ numbers in arithmetic progression with common difference d equal to $\frac{b-a}{n+1}$. Thus, the n arithmetic means in between a and b are $a+\frac{b-a}{n+1}$, $a+2\left(\frac{b-a}{n+1}\right)$, ..., $a+n\left(\frac{b-a}{n+1}\right)$. The **arithmetic mean** of a and b is taken as $\frac{a+b}{2}$.

The arithmetic mean of n different quantities, $a_1, a_2 \cdots, a_n$ is $\dfrac{a_1+a_2+\cdots+a_n}{n}$.

Arithmetic Progression (arithmetic sequence) (A.P.). A sequence in which the difference between any two consecutive terms is constant. The constant difference is called common difference. If a is the first term and d the common difference of an A.P. then its nth term is equal to $a+(n-1)\,d$, and sum of its n terms, S_n, is given by

$$S_n=\frac{n}{2}\{2a+(n-1)d\}.$$

Arithmetic Series. A series formed with the terms of an arithmetic progression. For example, the series $2+5+8+11+14+\cdots$ is an arithmetic series with first term 2 and common difference 3.

Arithmetico-Geometric Series. A series whose rth term is the product of the rth term of an A.P. and the rth term of a G.P. The rth term is $[a+(r-1)d]\,x^{r-1}$. If S_n denotes the sum of its first n terms, then

$$S_n=a+(a+d)x+(a+2d)x^2+\cdots+[a+(n-1)d]x^{n-1} \quad \cdots(1)$$

$$xS_n=a.x+(a+d)x^2+\cdots+[a+(n-2)d]x^{n-1}+[a+(n-1)d]x^n \quad \cdots(2)$$

Substracting (2) from (1) gives

$$S_n(1-x)=a+dx+d\,x^2+\cdots+dx^{n-1}-[a+(n-1)d]x^n$$

$$=a-[a+(n-1)d]\,x^n+\frac{dx(1-x^{n-1})}{1-x}$$

or $$S_n=\frac{a-\{a+(n-1)d\}x^n}{1-x}+\frac{dx(1-x^{n-1})}{(1-x)^2}.$$

Arm. Syn. *Side of an angle.* One of the lines forming a given angle.

Arm of a Couple. The shortest distance between the two parallel forces forming a couple. See *Couple.*

Array. An ordered arrangement of numbers or some other objects, used in determinants and matrices.

Ascending Order. A sequence or a series is said to be in ascending order if the power of any term is greater than the power of its preceding term.

Assembler. A special program or a compiler which converts the assembly language to machine language. See *Program*

Assembly Language. A machine oriented low level language. See *Program.*

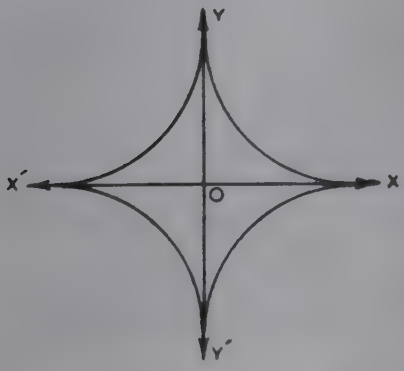
Associative Law. A property showing that an operation is independent of the mode of grouping the elements. Thus, a binary operation * is said to be associative if for any three elements a, b, c of a set

$a*(b*c)=(a*b)*c$. For example, the operations of addition, multiplication are associative over the set of numbers, while subtraction and divison operations are not associative.

Astroid. A hypocycloid with four cusps.

For example, the curve

$$x^{2/3}+y^{2/3}=a^{2/3}.$$



Astronomical Unit (A.U.). The mean distance between the sun and the earth. It is approximately 1.495×10^8 Km.

Astronomy. The branch of mathematical science in which we study the position and movement of celestial objects, the sun, planets and satellites, comets, etc. of the solar system and stars of galaxies.

Asymmetry. Opposite to symmetry. See *Symmetry.*

Asymptote. Tangent to a curve at infinity. A line which touches a given curve and the point of contact is at infinity, is called asymptote to the curve. For example, both the coordinate axes are asymptotes to the rectangular hyperbola $xy=c^2$.

Atmospheric Pressure. The pressure at a point due to the weight of air above that point. It is equal to the weight of a column of water about 30 ft. high or of mercury 760.1 mm high. Its' value is around 100 k Pa (100000 newtons per square meters). The atmospheric pressure decreases with elevation above sea-level and with increasing humidity of the atmosphere.

At Par. See *Premium.*

Attraction. See *Gravitation.*

Auxiliary Circle. The auxiliary circle of ellipse is the concentric circle having the major axis as diameter. The auxiliary circle of hyperbola is the concentric circle having the transverse axis as diameter.

Auxiliary Equation. See *Differential Equation,*

AU. See *Astronomical unit.*

Average. Syn. *Mean.* See *Arithmetic means, Geometric means, and Harmonic means.*

Average Speed. See *Speed.*

Axes of Coordinates. See *Cartesian coordinates.*

Axes of Ellipse. In the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ $(a>b)$ the diameter of length $2a$ passing through the foci is the *major axis* and that of length $2b$ at right angles to the major axis is the *minor axis.*

Axes of Ellipsoid. In the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ $a>b>c)$ the diameter of length $2a$ passing through the foci is

the *major axis*, that of length $2b$ at right angles to the major axis is *mean axis*, and that of length $2c$ perpendicular to both the major and mean axis is the *minor axis*.

Axes of Hyperbola. In the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, ($a > b$) the diameter of length $2a$ which when extended passes through the foci is the *transverse axis* and the line segment of length $2b$ through the centre and perpendicular to the transverse axis is the *conjugate axis*.

Axes of Hyperboloid. A hyperboloid has three axes, out of these those which are diameters are called *transverse axes* and those which are not diameters, are called *conjugate axes*. A hyperboloid of one sheet has two transverse axes and one conjugate axis while the hyperboloid of two sheets has one transverse axis and two conjugate axes.

Axial Plane. Syn. *Coordinate planes*. In three dimensional coordinate geometry the planes $x=0$, $y=0$, and $z=0$ are called axial planes

Axial Symmetry. Symmetry about a line.

Axiom. Syn. *Postulate*. An assumption in mathematical or logical system which is accepted as true without proof and from which further results in the form of lemmas or theorems can be derived.

Axis. 1. A line of symmetry in a given figure.

2. A fixed reference line used in a coordinate system.

3. A line about which a body or a curve may be considered to rotate or revolve. For example, the centre line of a right circular cone.

Axis of Earth. Axis of rotation of the earth, coinciding with that diameter which joins the geographical north and south poles.

Axis of Symmetry. A line about which a figure is symmetrical.

Azimuth. A coordinate to describe the position of a point on the celestial sphere. The angle measured in a horizontal plane from the x-axis in spherical polar coordinates. It is comparable with the longitude of a point on earth.

B

Ball. See *Sphere.*

Ballistic Pendulum. A heavy pendulum used to measure the velocity of a projectile.

Ballistics. The branch of mathematics that deals with the motion of projectiles.

Banach Algebra. An algebra over a field of real or complex numbers for which $\| xy \| \leqslant \| x \| \, \| y \|$ for all x and y. It is called a real or complex Banach algebra according as the field is real or complex number field. For example, the set of all functions which are continuous on the closed interval [0, 1] is a Banach Algebra over the field of real numbers if $\| f \|$ is defined to be largest value of $| f(x) |$ for $0 \leqslant x \leqslant 1$.

Banach Space. A vector space whose scalar multiples are the real numbers (or the complex numbers) and which has associated with each element x a real number $\| x \|$, the norm of x, satisfying the postulates :

(1) $\| x \| > 0$ if $x \neq 0$;

(2) $\| ax \| = | a | . \| x \|$ for all a and x ;

(3) $\| x+y \| \leqslant \| x \| + \| y \|$ for all x and y ;

(4) the space is complete, that is, a neighbourhood of the element x being the set of all y satisfying $\| x-y \| < \epsilon$ for

some fixed ϵ. Without postulate (4) the space is a normed linear space or normed vector space.

Bar. Unit of pressure in C.G.S. System.

$$1 \text{ Bar} = 10^6 \text{ dynes/cm}^2.$$

Bar Graph. A mode of representation of statistical data. The data is represented by vertical bars of equal widths, their lengths depending upon the frequencies.

Barn. Unit of area for measuring small areas.

$$1 \text{ Barn} = 10^{-24} \text{ cm}^2.$$

Base. A side (or a face) of a geometric configuration, upon which an altitude is constructed or is thought of as being constructed. For an expression such as a^n, the quantity a is called the base and n the exponent.

Base of a Logarithmic System. See *Logarithm.*

Basic. A high level Computer language.

Basis of a Vector Space. A set of linearly independent vectors such that every vector of the space is equal to some finite linear combination of vectors of the basis.

Bayes' Theorem. Suppose A and $B_1, B_2, \ldots, B_n$ are events for which the probability $P(A)$ of A is not zero, $\sum_{i=1}^{n} P(B_i) = 1$, and $P(B_i \text{ and } B_j) = 0$ if $i \neq j$. Then the conditional probability $P(B_j/A)$ of B_j given that A has occurred, is given by

$$P(B_j/A) = \frac{P(B_j)\, P(A/B_j)}{\sum_{i=1}^{n} \{P(B_i)\, P(A/B_i)\}}.$$

Bei Functions. See *Ber Functions.*

Below Par. See *Premium.*

Bending Moment. The algebraic sum of the moments of all external forces acting on one side of a vertical section of a beam.

Ber Functions. The Ber and Bei functions are given by

$$\text{ber}_n(z) \pm i\, \text{bei}_n(z) = J_n(z\ e^{\pm 3\pi i/4})$$

where J_n is a Bessel function. See *Bessel Function.*

Bernoulli Equation. A differential equation of the form

$$\frac{dy}{dx} + y f(x) = y^n g(x).$$

Bernoulli Trial. An experiment having two mutually exclusive outcomes.

Bessel's Differential Equation. The differential equation

$$x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} + (x^2 - n^2)\, y = 0.$$

Bessel Functions. The n^{th} order Bessel function, n being integer, is the coefficient of t^n in the expansion of $e^{x[t-(1/t)]/2}$ in powers of t and $\frac{1}{t}$. In general

$$J_n(x) = \frac{1}{\pi} \int_0^{\pi} \cos(nt - x \sin nt)\, dt$$

$$= \sum_{r=0}^{\infty} \frac{(-1)^r}{r!\, \Gamma\, n+r+1} \left(\frac{x}{2}\right)^{n+2r}$$

the second form being valid for $x \neq -1, -2, \ldots$. Here $J_n(x)$ is the solution of first kind of Bessels' differential equation. See *Bessels' Differential Equation.*

Beta Function. The function β defined by

$$\beta(m, n) = \int_0^1 x^{m-1} (1-x)^{n-1}\, dx$$

where m and n are positive. In terms of Gamma function Γ

$$\beta(m, n)=\frac{\lfloor\overline{m}\ \lfloor\overline{n}}{\lfloor\overline{m+n}}.$$

Biannual. Twice in a year.

Biconditional. See *Algebra of Propositions.*

Biennial. Once in two years or semi annual.

Biharmonic Equation. See *Biharmonic Function.*

Biharmonic Function. A solution of the fourth order partial differential equation $\nabla^4 u=0$

where $$\nabla^2\equiv\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}$$

The biharmonic functions occur in the study of elastostatic boundary value problems and elsewhere in mathematical physics.

Bijection. A one-one correspondence between two sets A and B, that is, both *injection* and *surjection.*

Billion. In Britain and Germany it is one million millions ; where as in U.S.A. and France it is one thousand millions.

Bimodal. A statistical distributlon having two modes. See *Mode.*

Bilinear. A mathematical expression which is linear with respect to each of the two variables concerned. A function $f(x, y)$ is bilinear if

$$f(x_1+x_2, y)=f(x_1, y)+f(x_2, y)$$

and $$f(x, y_1+y_2)=f(x, y_1)+f(x, y_2).$$

Binary Number System. A system of numerals for representing real numbers that uses base 2 instead of usual base 10. Here only two digits 0 and 1 are needed. For example, the number 101110 in binary notation is equal to

$$1.2^5+0.2^4+1.2^3+1.2^2+1.2^1+0.2^0$$

or 46 in decimal notation. The reverse process is accomplished by successive division by 2. For example, 29 in decimal notation is equal to

$$2.14+1=2^2.7+1=2^2(2.3+1)+1=2^3.3+2^2+1$$
$$=2^3(2+1)+2^2+1=2^4+2^3+2^2+1$$

which is equivalent to 11101 in binary notation. Similarly, for rational numbers. For example, .11 in binary notation is equal to $\frac{1}{2}+\frac{1}{2^2}=3/4$ in decimal notation. Binary arithmetic is very useful in electronic computers since the digits 0 and 1 can be described electrically as 'off' and 'on' stages of a switch.

Binary Operation. An operation applied to two objects. The addition of two numbers, multiplication of two numbers, intersection of two sets, product of two matrices are all examples of binary operations. A set S is said to be closed with respect to a particular binary operation * if for any $a, b \in S$, $a * b \in S$.

Binomial. A polynomial having two terms, *e.g.*, $3x+4y$.

Binomial Coefficients. The coefficients in the expansion of $(x+a)^n$. The $(r+1)^{th}$ binomial coefficient of order n is equal to $n!/[r!\,(n-r)!]$ *i.e.* the number of combinations of n things, taken r at a time. It is denoted by ${}^nC_r$ or (C_r) or $C(n, r)$. Sum of the binomial coefficients is equal to 2^n.

Binomial Distribution. A random variable x follows binomial distribution if there is an integer n and a number p such that x is the number of successes in n independent trials, where the probability of success in a single trial is p. The probability of k successes $(0 \leqslant k \leqslant n)$ is

$$P(x=k)={}^nC_k\, p^k \,.\, q^{n-k}$$

where $q=1-p$.

For example, if two coins are tossed then $p=q=\frac{1}{2}$ and the possibilities of 0, 1, 2 heads are $\frac{1}{4}$, $\frac{1}{2}$, $\frac{1}{4}$ respectively and these are the terms in the expansion of $\left(\frac{1}{2}+\frac{1}{2}\right)^2$ by binomial theorem. In general,

$$(p+q)^n=\sum_{k=0}^{n} {}^nC_k\, p^k\, q^{n-k}=\sum_{k=0}^{n} P(x=k).$$

The mean of the binomial distribution is np, the variance npq, and the moment generating function is $M(t)=(q+pe^t)^n$. When n is large the Binomial distribution may be approximated by a normal distribution with mean np and variance npq.

Binomial Equation. An equation of the form $a^n-b=0$.

Binomial Expansion. The expansion given by binomial theorem. See *Binomial Theorem.*

Binomial Series. The expansion of $(x+y)^n$ in the form of an infinite series where n is not a positive integer or zero. Such an expansion in infinite series converges to $(x+y)^n$ if $|y|<|x|$ For example,

$$\sqrt{3}=(2+1)^{1/2}=2^{1/2}\left(1+\frac{1}{2}\right)^{1/2}$$

$$=2^{1/2}\left[1+\frac{1}{2}\cdot\frac{1}{2}+\frac{\frac{1}{2}\left(\frac{1}{2}-1\right)}{1.2}\frac{1}{2^2}\right.$$

$$\left.+\frac{\frac{1}{2}\left(\frac{1}{2}-1\right)\left(\frac{1}{2}-2\right)}{1.2.3}\frac{1}{2^3}+..\right]$$

$$=2^{1/2}+\frac{1}{2}\cdot 2^{-1/2}+\frac{1}{2^3}\cdot 2^{-3/2}+\frac{1}{2^4}\,2^{-5/2}....$$

Binomial Theorem. A theorem for the expansion of a power of a binomial. This theorem states : The first term in the expansion of $(x+y)^n$ is x^n ; the second term has n for its coefficient and the other factors are x^{n-1} and y ; in subsequent terms the power of x decreases by 1 for each term and that of y increases by 1 while the next coefficient is obtained from a given coefficient by multiplying by the exponent of x and dividing by a number one more than exponent of y.

Thus, if x is a positive integer,

$$(x+y)^n=x^n+nx^{n-1}y+\frac{n(n-1)}{2!}x^{n-2}y^2+\frac{n(n-1)(n-2)}{3!}x^{n-3}y^3$$

$$+...+y^n.$$

The coefficient of $x^{n-r}y^r$ is $\frac{n!}{r!(n-r)!}$. The binom

theorem holds good for any exponent whatsoever but und certain restrictions on the value of x and y. See *Binom series.*

Biquadratic Equation. Syn. *Quartic.* Any algebr equation of fourth degree.

Birectangular triangle. Any spherical triangle two whose angles are right angles. See *Spherical Triangle.*

Bisect. To divide in half.

Bisector. A straight line or a plane that divides a line o plane, or an angle into two equal parts. The equation of the l bisecting a given angle, is obtained by equating the perpendi lar distances from a variable point to each of the lines formi the angle. The equations of the planes bisecting the an between two given planes, are obtained by equating the perp dicular distances from the variable point from the two giv planes first giving these distances like signs and then unl signs.

Bit. Abbreviation of binary digit, that is, either of digits 0 and 1 which constitue the binary number system. E are the basic units of digital information to the comput These bits 0 and 1 are equivalent to the electrical 'on' and 'o See *Binary Number System.*

Bivariate. Involving two variable quantities.

Bolzano's Theorem. A single real valued function f of a real variable x, is zero for at least one valus of x on $[a,$ if it is continuous on $[a, b]$ and $f(a)$ and $f(b)$ have oppo signs.

Bolzano-Weierstrass Theorem. If E is a bounded containing infinitely many points, there is a point x which an accumulation (limit) point of E. The set E may be a of real numbers, a set in a plane, or a set in n-dimensio Euclidean space. Weierstrass gave this theorem while Bolza proved it.

Boolean Algebra. A system of mathematical logic which uses symbols and set theory to represent logical operations in mathematical form. After this system many more systems have been developed and are used in the probability theory and computing.

Bound of a Function. Bound of the set of all possible values of the function of a variable. For example, the trigonometric functions sine and cosine are bounded by $+1$ (the l.u.b.) and -1 (g.l b.) The exponential function 3^x is bounded by 0 (g.l.b.) as $x \to -\infty$, and is unbounded as $x \to \infty$. See *Bound of a Set.*

Bound of a Set. A number that is greater than or equal to every member of the set, is an *upper bound* of the set, and the number that is less than or equal to every member of the set is called a *lower bound* of the set. *Least upper bound axiom* : "A set of real numbers that has an upper bound, has a least upper bound". Greatest lower bound axiom : "A set of real numbers that has a lower bound, has a least lower bound".

Boundary Condition. In a differential equation the values of the variables at a certain point or information about their relationship at a point, is used in evaluating the arbitrary constants. For example, the differential equation

$$\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 0$$

has a general solution as $y = Ae^{2x} + Be^{3x}$ where A and B are arbitrary constants. If the boundary condition are $y=0$ and $\frac{dy}{dx} = 1$ at $x=0$ then $A = -1$, $B=1$.

Boundary Value Problems. The problems of finding a solution to a given differential equation or a set of differential equations, which will meet certain specified requirements for a given set of values of the independent variables—the boundary points. Many of the problems of mathematical physics are of this type.

Breadth. Syn. *Width.* The horizontal distance, usually taken at right angles to the length

Briggsian Logarithim. Syn. *Common Logarithm.* A logarithm with base 10. See *Logorithm.*

British Thermal Unit (B. t. u.) The amount of heat energy required to raise the temperature of 1 lb. of water from 60°F to 61°F. It is equivalent to 1055.06 Joules.

Brokerage. Commission paid for financial transactions.

Bug. An error or fault in the compilation or execution of a computer program.

Byte. Number of bits used to represent a constant or a variable in a program. In most of the computers a byte consists of a fixed number of bits, generally eight. See *Bit.*

C

Calculation. The process of obtaining results from the given data by using mathematical procedures. Earlier the term used to imply a numerical result, but it is now used as a general term for any procedure which produces a required result numerical or otherwise. *Computation* is the term being used now to mean numerical calculation.

Calculus. The branch of mathematics which deals with differentration and integration of functions and related concepts and applications. Also called infinitesimal calculus because of the prominence of the use of infinitesimals in the early development of the subject. See *Infinitesimal.*

Calculus of Variations. The study of the theory of maxima and minima of definite integrals whose integral is a known function of one or more independent variables and of one or more dependent variables and their derivatives. The problem is to determine the dependent variables so that the integral will be a maxima or a minima. The simplest of such integral is of the form

$$I=\int_a^b f\left(x, y, \frac{dy}{dx}\right) dx$$

where y is to be determined so as to make I a maximum or a minimum (whichever is desired). It was initiated by Lagrange.

Calendar Year. See *Year.*

Calorie. The amount of heat required to raise the temperature of one gram of water by one degree centigrade. The calorie thus defined varies slightly for different temperatures. A standard calorie is usually defined as the amount of heat required to raise the temperature of one gram of water from 14.5°C to 15.5°C. It equals 4.1840 absolute Joules.

Cancellation. The act of dividing by like factors the numerator and denominator of a fraction ; sometimes used for two quantities of different signs which cancel each other in addition.

Canonical (normal) Form. The diagonal matrix obtained by applying a number of transformation on another square matrix of the same order. See *Matrix.*

Cantilever. A projecting beam supported at one end only, girder or bracket.

Cap. (1) The symbol $\cap$ used to denote the intersection of two sets. See *Intersection.* (2) In connection with a sphere, it is the smaller portion of the sphere when it is intersected by a plane other than that through its centre.

Card. Syn. *Punch Card.* A rectangular piece of quality paper of size 18.73×8.25 cm. A punch card is the most commonly used and cheap means of feeding information and data into a computer. Information is recorded as a pattern of rectangular

holes in the cards. A card has 80 columns and 12 rows in which holes are punched. The first row is numbered 'Y' or '12', second row as 'X' or '11', and other rows are marked 0 to 9 from top to bottom. Cards are punched on a punching machine which is quite similar to a typewriter. Pressing a key results in a rectangular hole in a particular column. Hole or combination of holes in a column represent different characters. A single hole punched in any of 0 to 9 rows represents a digit from 0 to 9 respectively whereas an alphabet needs two holes one in 'X', 'Y' or 0 rows and second in0—9 rows. For a special character a combination of three holes is there in a column. There is a provision for getting the characters printed also at the top of the card.

Cardinal Number. A whole number which specifies how many elements are there in a set.

Cardioid. An epicycloid having only one loop. It is a special case of *lamicon.* Cardioid is the path traced out by a point on a circle rolling round the circumference of another circle having the same radius. Its equation is $r=2a\,(1\pm\cos\theta)$. See *Epicycloid, Lamicon.*

Cardon's solution of the Cubic. A solution of the reduced cubic $x^3+ax+b=0$ (obtained from the general cubic $a_0x^3+a_1x^2+a_2x+a_3=0$ by putting $x=x-a_1/a_0$) by the substitution $x=u+v$ [$x=u+v$ will be a root of the equation if $u^3+v^3=-b$ and $uv=-a/3$, that is if u^3 is a root of the quadratic

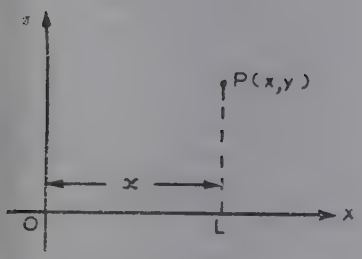
$(u^3)^2+bu^3-a^3/27=0$, and $uv=-a/3$]

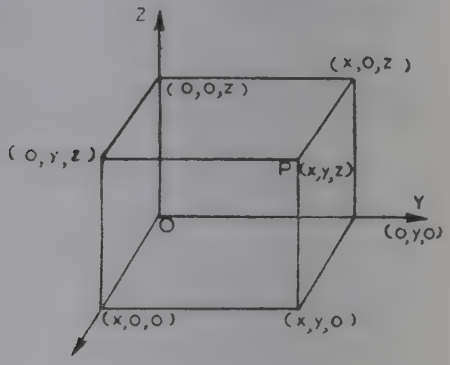
Capacity. Internal volume, *i.e.*, the amount of space enclosed. Its units are litre or gallon. It is also used in electricity to denote the charge on a capacitance.

Cartesian Coordinates. These are named after the French mathematician Descartes who used these for studying *analytical geometry.* This gives a method of defining the position of a point by its distance from a fixed point (origin) in the direction of two or more straight lines. In a plane two straight lines called the x-axis and y-axis form the basis of a two dimensional coordinate system. The point of intersection of the two axes is called the origin. An imaginary grid is formed

by lines parallel to the axes and unit distance apart. For example, the point (3, 5) is the point where the line parallel to y-axis three units away from y-axis, meets the line parallel to x-axis five units away from x-axis.

The position of a point in space can be determined by three measurements with respect to three concurrent lines called *axes of coordinates*. These axes form a frame of reference and the point of concurrency is the origin of coordinates. The axes are referred to as x-axis, y-axis, and z-axis, and the coordi-





nates of a point are (x, y, z) as shown in the figure. The axes are usually taken as mutually perpendicular and hence are called *rectangular axes* and the coordinates called *rectangular coordinates*. When the axes are not at right angles these are called *oblique axes*.

Cartesian Equation. An equation of a configuration in cartesian coordinates. See *Cartesian Coordinates*.

Cartesian Product. The set of all ordered pairs of elements (a, b) where $a \in A$ and $b \in B$ is called cartesian product of two sets A and B, written as $A \times B$.

Casting out Nines. A method of checking the accuracy of an operation on integers by applying the same operation to the sums of the digits of the integral numbers when these sums have been written in modulo 9 number system. This is

called *excess of nines*, the remainder when a number is divided by 9. For example,

addition : to check $652+745=1397$. The sum of the digits for the three numbers are 13, 16, and 20. In modulo 9, $13+16=4+7=11=2 \pmod 9$ and $20=2 \pmod 9$.

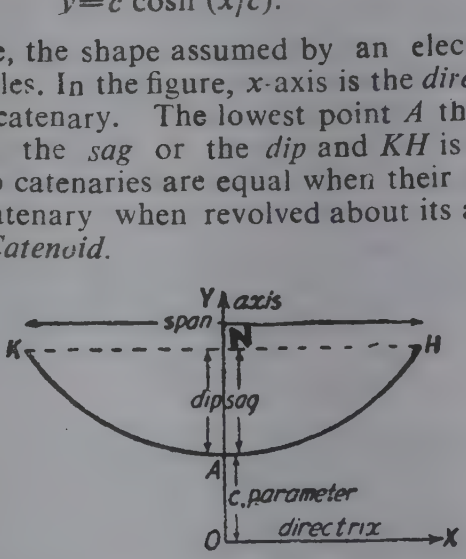
multiplication : To check $479\times516=247164$, the sum of the digits for the three numbers are 20, 12, 24. In modulo 9, $20\times12=2\times3=6 \pmod 9$ and $24=6 \pmod 9$.

The method is also applicable to the operations of subtraction and division involving integral quotients.

Catenary. Syn. *Chainette*. When a uniform flexible chain freely hangs from its two extremes the shape of the curve assumed by the chain is called catenary with equation as

$$y=c\cosh(x/c).$$

For example, the shape assumed by an electric wire hung between two poles. In the figure, x-axis is the *directrix* and y-axis the *axis* of the catenary. The lowest point A the *vertex*, c the parameter, NA the *sag* or the *dip* and KH is the *span* of the catenary. Two catenaries are equal when their parameters are equal. The catenary when revolved about its axis generates a surface called *Catenoid*.



Cauchy–Riemann Equations. Syn. *C—R Equations*. A necessary condition for a function of complex variables to be analytic. A function $f(z)=u+iv$, [where $z=x+iy$, $u=u(x, y)$ and $v=v(x, y)$] is analytic if

$$\frac{\partial u}{\partial x}=\frac{\partial v}{\partial y},\quad \frac{\partial u}{\partial y}=-\frac{\partial v}{\partial x}.$$

These equations are called Cauchy—Riemann equations. The polar form of these equations are

$$\frac{\partial u}{\partial r}=\frac{1}{r}\frac{\partial v}{\partial \theta},\quad \frac{\partial v}{\partial r}=-\frac{1}{r}\frac{\partial u}{\partial \theta}.$$

See *Analytic.*

Cauchy's Convergence Criterion. A sequence $\{u_n\}$ converges if and only if, for each $\epsilon>0$, there exists a number N, such that $|u_p-u_q|<\epsilon$, for $p, q>N$. This criterion has the advantage that one need not know the limit to decide the convergence. See *Sequence, Convergence.*

Cauchy's form of Remainder. See *Taylor's Theorem.*

Cauchy's Generalized Theorem. If $f(x)$ and $g(x)$ are continuous in $[a, b]$ and differentiable in (a, b), then there exists a point ξ in (a, b) such that

$$\frac{f(b)-f(a)}{g(b)-g(a)}=\frac{f'(\xi)}{g'(\xi)},\ g(a)\neq g(b) \text{ and } f'(x) \text{ and } g'(x)$$

are not zero simultaneously.

Cauchy's Integral Formula. If $f(z)$ is analytic inside and on a closed contour C and $z=z_0$ is a point inside C then

$$\frac{1}{2\pi i}\int_C \frac{f(z)dz}{z-z_0}=f(z_0),$$

$$\frac{1}{2\pi i}\int_C \frac{f(z)dz}{(z-z_0)^2}=f'(z_0).$$ Continuing the process

$$\frac{n!}{2\pi i}\int_C \frac{f(z)dz}{(z-z_0)^{n+1}}=f^{(n)}(z_0).$$

Cauchy's Integral Test. If $f(x)$ is positive, continuous and monotonic decreasing for $x\geqslant N$ and is such that,

$$f(n)=u_n,\ n=N,\ N+1,\ N+2,\cdots,$$

then $\sum_{n=1}^{\infty} u_n$, converges or diverges, according as

$\int_N^{\infty} f(x)\,dx = \underset{M\to\infty}{\text{Lt}} \int_N^M f(x)\,dx$ converges or diverges. In practice, we take $N=1$. For example $\sum_{n=1}^{\infty} \frac{1}{n^3}$ converges because

$$\underset{M\to\infty}{\text{Lt}} \int_1^M \frac{dx}{x^3} = \underset{M\to\infty}{\text{Lt}} \left(1-\frac{1}{2M^2}\right) \text{ exists finitely.}$$

Cauchy's Residue Theorem. If $f(z)$ is analytic within and on a closed curved C except at a finite number of points $z=z_1, z_2, \cdots, z_n$ inside C then

$$\int_C f(z)dz = 2\pi i \sum_{r=1}^{n} \text{Res}\, f(z_r)$$

Cauchy's Root Test. A positive term series $\sum_{n=1}^{\infty} u_n$, converges if $\underset{n\to\infty}{\text{Lt}}\ (u_n)^{1/n}<1$ and diverges if $\underset{n\to\infty}{\text{Lt}}\ (u_n)^{1/n}>1$. The test fails to give any information if $\underset{n\to\infty}{\text{Lt}}\ (u_n)^{1/n}=1$. For example the series $\frac{2}{3}+\left(\frac{2}{3}\right)^2+\left(\frac{2}{3}\right)^3+\ldots\infty$, converges, because $u_n=\left(\frac{2}{3}\right)^n$ and $\underset{n\to\infty}{\text{Lt}}\ (u_n)^{1/n}=\frac{2}{3}<1$. See *Series, Convergence.*

Cauchy's Theorem. If $f(z)$ is analytic inside and on a closed contour C then

$$\oint_C f(z)dz=0.$$

Celsius Grade. (Symbol °C). A scale of temperature named after the Swedish Physicist Celsius who based it on the freezing temperature of water (0°C) and the boiling point of water (100°C) as fixed extremes at normal atmospheric pressure.

The interval between these two temperatures is divided into 100 equal parts, each part being called 1°C.

Cent. Stands for 10^2.

Cent, in Percentage. See *Percent, Percentage.*

Centigrade Scale. Syn. *Celsius Grade.*

Central Conic. A conic section which has a centre of symmetry, *e.g.*, *ellipse* and *hyperbola.*

Central Force. A force that acts on an object in such a way that it is always directed towards a fixed central point.

Central Symmetry. Symmetry about a point.

Central Processing Unit (*Central Processor*). Denoted by C.P.U. It can be rightly called the nerve centre of a computer or the brain of it. It consists of a *control unit* and an *arithmetic and logical unit* (A.L.U.) The control unit controls all the activities inside the computer. The set of all instructions is called the *program.* Each instruction is brought in sequentially from the main store, and is kept for a while in a small store called *register.* The instruction is analysed and interpreted as to what operation is to be performed. The A.L.U. performs the operations actually through complex electronic circuits and a set of registers. The operation may be an arithmetic operation or a logical operation which requires decision making. The process of fetching the information, analysing it, and executing it is repeated in a specified order until an instruction to stop is executed.

The C.P.U. size has now been remarkably reduced, thanks to the latest technological advancements. It has become possible to form a central processor on a single silicon chip on a very small area. The device is called *microprocessor* and is very popular these days.

Centre. A point about which a geometrical figure is symmetrical.

Centre of Buoyancy (*with reference to a fluid*). The centre of mass of the liquid displaced by a floating body. The condition for the stability of a floating body is that the centre of mass of the object must lie vertically below the centre of buoyancy.

Centre of Curvature. See *Curvature.*

Centre of Gravity. The point in a body at which the whole weight of the body may be considered to be acting. In other words it may be thought of as a point through which the resultant of all the gravitational forces acting on all the particles of the body can be considered to be acting.

Centre of Mass. The point in a body at which the entire mass may be concentrated, theoritically, to produce the same effect as when distributed. The term is slightly different than centre of gravity. In uniform gravitational field centre of mass is the same as centre of gravity. The centre of mass coincides with the centre of symmetry if a symmetrical body has a uniform density throughout.

If (x_1, y_1, z_1), (x_2, y_2, z_2), ..., (x_n, y_n, z_n) are coordinates of masses $m_1, m_2, \ldots, m_n$ then the centre of mass $(\bar{x}, \bar{y}, \bar{z})$, is given by

$$\bar{x}=\frac{\sum_{r=1}^{n} m_r x_r}{\sum_{r=1}^{n} m_r}, \quad \bar{y}=\frac{\sum_{r=1}^{n} m_r y_r}{\sum_{r=1}^{n} m_r}, \quad \bar{z}=\frac{\sum_{r=1}^{n} m_r z_r}{\sum_{r=1}^{n} m_r}.$$

Centre of Pressure. The point in a body submerged in a liquid, at which the resultant of all the pressure forces may be concentrated, theoritically, so as to produce the same effect as when distributed throughout the body.

Centre of Regular Polygon. In the regular polygon the centre of the inscribed circle coincides with the centre of the circumscribed circle and the common point is termed as centre of polygon.

Centrifugal Acceleration. See *Centrifugal Force.*

Centrifugal Force. A force acting radially out ward on a body moving in a circle. If a body of mass m moves along a circularpath of radius r, with a uniform speed v, then the magnitude of this force is $\frac{mv^2}{r}$. The quantity $\frac{v^2}{r}$ is known as *Centrifugal Acceleration.*

Centripetal Acceleration. See *Centripetal Force.*

Centripetal Force. The force acting radially inward on a body moving in a circular path. This force is directed towards the centre of the circle. If v is the uniform speed and r the radius of the circle, then the magnitude of this force is $\frac{mv^2}{r}$, where m is the mass of the body. The radial acceleration of the body v^2/r, directed towards the centre is known as *Centripetal Acceleration.*

Centriod. The point in a plane figure or a solid at which the centre of mass would be if the body has uniform density throughout. Centroid of a symmetrical figure will be the centre of symmetry. For example, the centroid of a triangle is the point of intersection of its medians, centroid of circle or a sphere is its centre. See *Centre of Mass.*

C.G.S. System. A system of physical units, in which unit of mass is *gram*, unit of length is *centimetre* and unit of time is *second.* See *M.K.S. System also.*

Chain. An instrument used in surveying. It is of standard length 22 yards (66 ft.) and is convenient as 10 square chains is equal to 1 acre (4840 sq. yds.).

Chain Rule. A rule used for differentiating a function of a function. If z is a function of y and y is a functions of x then

$$\frac{dz}{dx} = \frac{dz}{dy} \cdot \frac{dy}{dx} .$$

The chain rule can be extended to functions of several variables also.

Character. A symbol that can be represented in a computer. There are in all 47 characters as follows.

letters	A—Z	26
numerals	0—9	10

other spmbo.s=, +, −, *, /, **, ., space, $, , (,)

A character is stored in a computer as a group of bits, *i.e.*, binary digits 0 and 1. See also *Bit*, *Byte*, *Word*.

The characters A—Z are called Alpha Characters and the digits 0—9, numeric characters. The two together are known as Alpha Numeric Characters.

Characterstic. The integral part of the logarithm of a number determined by the place value of the digit having the highest place value. Thus, unit numbers have characterstic 0, tens and tenths 1 and −1 respectively, hundreds and hundredths 2 and −2 respectively, thousands and thousandths 3 and −3 respectively, etc.

Chi-square Distribution. The distribution of the sum of the squares of random variables with standard normal distribution. For-example, if $x_1, x_2, \ldots, x_n$ are independent and each of these has normal distribution, then

$$\chi^2 = \sum_{i=1}^{n} x_i^2$$

has a chi square distribution with n degrees of freedom. It is denoted by χ^2_n. The mean of this distribution is n and variance is $2n$.

Chi-square Test. The test determines as to how well a theoritical probability distribution fits a given set of data. If o_i is the observed value and e_i is the expected value of the variable x_i then

$$\chi^2=\sum_{i=1}^{n}(o_i-e_i)^2/e_i$$

follows a chi-square distribution with n degrees of freedom. See *Chi-square Distribution.*

Chord. A straight line joining any two points on a curve.

Chord of Contact. The line joining the points of contact of a pair of tangents drawn from a point to a curve.

Circle. The plane figure obtained as a locus of a point whose distance from a fixed point is constant. The fixed point is called *centre* of the circle and the constant distance is called *radius* of the circle. The diameter of a circle is double its radius r, the circumference is $2\pi r$ and area πr^2. Equation of a circle with radius r and centre at the origin in cartesian coordinates, is $x^2+y^2=r^2$, whereas the general equation of a circle is $x^2+y^2+2gx+2fy+c=0$ whose centre is $(-g, -f)$ and radius is $\sqrt{g^2+f^2-c}$.

Circle of Curvature. See *Curvature.*

Circular Cone. A cone having circular base. See *Cone.*

Circular Cylinder. A cylinder with circular base. See *Cylinder.*

Circular Functions. Syn. *Trigonometric Functions.* The sine, cosine, tangent, cotangent, secant, and cosecant are circular functions which are periodic with period 2π. The circular functions sine, cosine, secant, and cosecant of angle $(\theta\pm 2n\pi)$ have values which repeat for $n=0, 1, 2, \ldots$ whereas tangent and cotangent of angle $(\theta\pm 2n\pi)$ have values which repeat for $n=0, \frac{1}{2}, 1$, etc.

Circular Measure of Angles. The measurment of an angle in radians. See *Radian.*

Circular Motion. Motion of an object in a circle. It is possible if a positive force acts towards the centre. If the object moves with uniform velocity v, in a circle of radius r, the angular velocity (ω) is $\frac{v}{r}$. There is always an acceleration equal to $\frac{v^2}{r}$ towards the centre of the circle. See *Centripetal Acceleration.*

Circumcentre. The centre of the circumscribed circle of a plane figure. For example circumcentre of a triangle.

Circumcircle. Circumscribed circle. For example circumcircle of a triangle.

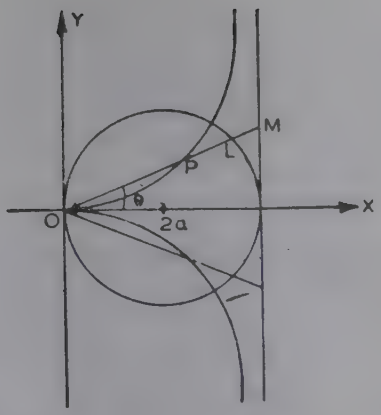
Circumference. The boundary, or the length of the boundary of a closed curve, usually a circle. For example, circumference of a circle equal to $2\pi r$ in length if r is the radius of the circle, *circumference of a sphere* is the circumference of any great circle of a sphere.

Circumscribed. A geometrical figure which is drawn around another figure in such a way that it just encloses the other figure. For example, *circumscribed circle of a polygon* is a circle which passes through all the vertices of the polygon ; *circumscribed circle of a triangle* has its centre at the point of intersection of the perpendicular bisectors of the sides of the triangle and its radius r is given by $r=\frac{abc}{4\Delta}$ where a, b, c are the length of the sides and Δ the area of the triangle.

Cissoid. A curve named after Diocles having polar equation as $r=\frac{2a\sin^2\theta}{\cos\theta}$ and cartesian equation as

$$y^2(2a-x)=x^3.$$

Let O be a fixed point on the circumference of circle of radius a. Draw any secant through O cutting the circle at L and the tangent at the extremity of the diameter through O, at



M. If *P* is a point on *OM* such that $OP=LM$ then the locus of he point *P* is the required cissoid with $x=2a$ as asymptote. See *Asymptote.*

Class. (1) Syn. Set (2) A group of data taken as one tem in a frequency distribution.

Class Interval. Difference between class limits. See *Class Limits.*

Class Limits. Limits of different classes in a frequency distribution. If the persons in a locality are classified into the age groups 0—5 years, 5—10 years, etc. then it should be clearly decided as to which group a child will be put, who is just 5 years old. Usually it is put in the class interval 5—10. In general in a class interval the upper limit is excluded. For example, the class interval 0—10, contains the quantities whose value is less than 10.

Clockwise. Rotating in the same direction as the hands of a clock. The anti-clockwise rotation is taken as positive and the clockwise as negative. For example, tightening of a screw is clockwise while the breaking the screw loose is anti-clockwise.

Closed Curve. (*closed contour*). A continuous curve whose end points are coincident. For example, a circle, an ellipse, etc. which forms a complete loop and has no end points. A *simple closed curve* is one which does not intersect itself.

Closed Interval. An interval which includes the two end points. For example, set of all real numbers greater than or equal to 0 and less than equal to 1, denoted by [0, 1].

Closed Set. A set which includes its all limit points. For example, set of all rational numbers x such that $2 \leqslant x \leqslant 3$, set of all points lying inside and on a circle.

Closed Surface. A continuous surface which does not have any boundary lines or boundary curves. For example, a sphere, polyhedra, ellipsoid.

Closed System. Suppose * is a binary operation. A system is said to be closed with respect to * if $a*b$ always belongs to the system for any two elements a and b belonging to the system.

Closure. The property which when satisfied by the elements of a system makes it a closed one. See *Closed System.*

Cluster Point. Syn. *Accumulation Point.*

Coaxial Circles. A system of circles which have a common radical axis. For instance, system of circles with centres on the x-axis and radical axis as y-axis, equation of such system being $x^2+y^2+2gx+c=0$, where c is arbitrary constant.

COBOL. Short form of COMMON BUSINESS ORIENTED LANGUAGE. It is a computer programing language used mainly for business and accountancy purposes.

Coding. Writing the instructions pertaining to the solution of a problem sequentially in a computer programing language. Coding is done with the help of a prepared flowchart. See *Flow Chart.*

Coefficient. A multiplying factor. Thus, in $3kxy$, $3kx$ is the coefficient of y, $3ky$ is the coefficient of x, and $3k$ is the coefficient of xy.

Coefficient of Correlation. In the given data if the variables x and y are related to each other, then the coefficient of correlation denoted by ρ or r is the quotient of $\Sigma(xy)$ and $\sqrt{\Sigma x^2\, \Sigma y^2}$. Thus, the measure of correlation between the variables x and y is given by

ρ (or r)$=\frac{\Sigma(xy)}{n\,\sigma_x\,\sigma_y}$, where σ_x and σ_y are the standard deviations of x and y respectively and n is the number of pairs of values of x and y. The coefficient of correlation always lies between -1 and 1. See *Standard Deviation.*

Coefficient of Friction. See *Friction.*

Coefficients, Differential. See *Differential Coefficients.*

Cofactor. Minor of a determinant alongwith proper sign attached. See *Minor of a Determinant.*

Coincident. Identical in position.

Collinear Planes. Syn. *Coaixal Planes.* Planes having a common straight line. The set of planes having a common straight line called the axis, forms a *pencil of planes.*

Collinear Points. Points lying on a straight line.

Column Matrix. See *Column Vector.*

Column Vector. Syn. *Column Matrix.* An arrangement of numbers in a column. If there are n elements, the column vector is said to be of order $n\times 1$.

Combination. If out of n given things, r things are to be selected at random, it can be done in $\frac{n\,!}{r\,!\,(n-r)\,!}$ ways which is denoted by ${}^nC_r$ where the letter C stands for combinations. Other notations used are $\binom{n}{r}$, $C\,(n, r)$, C_r. Here

$${}^nC_0={}^nC_n=1$$

and ${}^nC_0+{}^nC_1+{}^nC_2+\ldots\ldots+{}^nC_n=2^n$. See *Factorial* and also *Permutations.*

Combinatorial Analysis. Analysis of combinations. This involves the study of permutations, combinations, binomial theorem, magic squares, theory of partitions.

Commensurable. Measurable in terms of same units. For example, a 20 centimeter length is commensurable with 1 meter length as both can be measured in centimeters, but the length of a side and that of a diagonal of a square are not commensurable.

Common Denominator. A positive integer which is divisible by the denominators of two or more fractions. For example, 12 and 24 both are common denominators of the fractions $\frac{1}{3}$ and $\frac{1}{4}$ of which 12 is the lowest (*least*) *common denominator* (LCD).

Common Difference. The difference between any two consecutive terms of an arithmetic progression. See *Arithmetic Progression.*

Common Factor. A number or an expression which can divide two or more given numbers or expressions.

Common Fraction. See *Fractions.*

Common Logarithm. See *Logarithm.*

Common Multiple. A whole number which is a multiple of a group of numbers. For example, 50 is a common multiple of 2, 5, 10, 25. The lowest common multiple (L.C.M.) is the smallest of all the common multiples.

Common Ratio. In a geometric progression or in a geometric series the ratio of any term to its preceeding term.

Common Tangent. A line which is a tangent to two or more curves.

Commutative. A binary operation* is said to be commutative on a set S if for any $a, b \in S$ $a*b=b*a$. For example, the operation of addition and multiplication are commutative on set of real numbers.

Commutative Group. Syn. *Abelian Group.* A group in which the binary operation is commutative. See *Commutative* and *Group.*

Commutative Ring. A ring in which the operation of multiplication is commutative. See *Ring.*

Comparison Test. A test for convergence and divergence of an infinite series. If from and after certain term absolute value of each term of the series under test is less than or equal to the corresponding term of some convergent series, the given series is convergent. Similarly, if from and after certain term absolute value of each term of the series under test is greater than or equal to corresponding term of a divergent series, the given series is divergent.

In other words, if $\underset{n\to\infty}{\text{Lt.}} \frac{u_n}{v_n}$, is non-zero and finite then the series Σu_n behaves as the known series Σv_n does. The series Σv_n is generally taken as $\sum \frac{1}{n^p}$ which is already known to be convergent if $p>1$ and divergent if $p\leqslant 1$. See *Convergence, Divergence of Series.*

Compiler. See *Program.*

Complement. See *Set Theory.*

Complementary Angle. A pair of angles whose sum is a right angle $\left(90^\circ \text{ or } \frac{\pi}{2}\right)$. One angle of the pair is complement of the other.

Completing the Square. A method usually employed for factorising a quadratic expression or solving a quadratic equation. The process involves adding of a quantity to a given quantity so that the sum is a perfect square. To a quantity x^2, a square, is added another quantity $y(2x+y)$, a rectangle, to produce a square $(x+y)^2$. For example the quadratic equation $ax^2+bx+c=0$ may be solved as

$$x^2+\frac{b}{2a}x+\frac{c}{a}=0 \text{ or } \left(x+\frac{b}{2a}\right)^2-\frac{b^2}{4a^2}+\frac{c}{a}=0$$

or $$\left(x+\frac{b}{2a}\right)^2-\left(\frac{\sqrt{b^2-4ac}}{2a}\right)=0$$

or $$\left[x+\frac{b}{2a}+\frac{\sqrt{b^2-4ac}}{2a}\right]\left[x+\frac{b}{2a}-\frac{\sqrt{b^2-4ac}}{2a}\right]=0$$

which gives $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$.

See *Quadratic.*

Complex Fraction. A fraction having *a* fraction or frac tions in the numerator and or the denominator. For example the fractions $\dfrac{1/2}{4}$, $\dfrac{1}{3/4}$ and $\dfrac{1/2}{1/2}$.

Complex Number. A number of the form $z=x+iy$ where x and y are real numbers and $i=\sqrt{-1}$. Here x is called the real part of the complex number z, written as $Re\,(z)$ and the imaginary part of z, written as $Im\,(z)=y$.0 is also a complex number since $0=0+0.i$. To every complex number z there appears $\bar{z}=x-iy$ and $-z=-x-iy$.

Addition and subtraction.

$$(a_1+ib_1)\pm(a_2+ib_2)=(a_1\pm a_2)+i(b_1\pm b_2)$$

Multiplication. $(a_1+ib_1)(a_2+ib_2)$

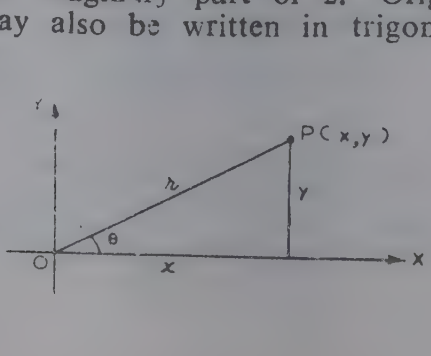
$$=(a_1a_2-b_1b_2)+i(a_1b_2+a_2b_1)$$

Division.

$$\frac{a_1+ib_1}{a_2+ib_2}=\frac{(a_1+ib_1)(a_2-ib_2)}{(a_2+ib_2)(a_2-ib_2)}$$

$$=\frac{(a_1a_2+b_1b_2)+i(a_2b_1-a_1b_2)}{a_2^2+b_2^2}$$

A complex number can always be represented by an Argand diagram. As shown in the figure the complex number $z\,(=x+iy)$ is represented by the point $(x,\ y)$. The horizontal axis is called real axis measuring real part of z, and the vertical axis the imaginary axis measuring the coefficient of i in z *i e.* the imaginary part of z. Origin O represents 0 $(0+i0)$. z may also be written in trigonometric form as



$e^{i\theta} = r(\cos\theta + i\sin\theta)$ so that $x = r\cos\theta$, $y = r\sin\theta$, r is called the modulus of z denoted by $|z| = \sqrt{x^2+y^2}$ and θ is called the argument or amplitude of z denoted by $\text{Arg.}(z) = \tan^{-1}(y/x)$.

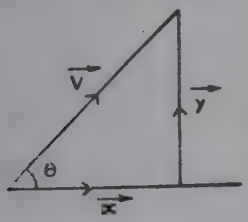
Componendo. See *Proportion.*

Component Forces. See *Component Vectors*

Component of a Vector. Component of a given vector $\vec{V}$ in a specified direction is the projection of the vector in that direction, *i.e.*, $\vec{V}\cos\theta$, where θ is the angle between the vector and that direction. See *Vector.*

Component Vectors. The componets of a vector, such as a force, displacement, velocity, are two or more vectors whose net effect is the same as that of the given vector, so that the given vector can be taken as the resultant of the component vectors. The mostly used component vectors are the pairs at right angles to each other say the pair (x, y) where

$$\vec{x} = \vec{V}\cos\theta, \quad \vec{y} = \vec{V}\sin\theta,$$



for any value of θ. See *Vector.*

Component Velocities. See *Component Vectors.*

Composite Number. Any integer having more than one prime factors. All composite numbers can be expressed as products of unique sets of prime numbers. This fact is also referred to as the fundamental theorem of arithmetic or the unique factorization theorem.

Composite of Functions. If f is a function which maps a set X into a set Y so that $y = f(x)$ for $x \in X$, $y \in Y$, and g is a function which maps the set Y into a set Z so that $z = g(y)$ for

$y \in Y$, $z \in Z$ then $g.f$ is a composite of the function g with t function which maps in set X into the set Z.

Composite of Relation. If R is a relation between t sets X and Y such that xRy for $x \in X$, $y \in Y$, and S is a relati between the sets Y and Z such that yRz for $y \in Y$, $z \in Z$, th the composite of S with R is a relation $S*R$. For example, t composite of (x is not equal to y) and (y is not equal to z) (x is not equal to z).

Composition of Vectors. Two vectors acting at a poi can be combined by the rule of *parallelogram of vectors* or t *triangle of vectors*. Three or more vectors can also be combin by the rule of *polygon of vectors*. See *Vector Sum*, *Vect Difference*.

Compound Interest. The interest earned on capital wh the interest is periodically added to the principal. The formu for calculating the final amount A when the interest has be added to the principal n times is $A = P\left(1 + \frac{r}{100}\right)^n$ where P the starting principal and r the rate of the compound interes The compound interest for any period is the difference of A a P.

Computation. The process of obtaining numerical resul from data by applying arithmetic operations or using slide rul logarithms, desk calculator, or the electronic computers.

Computer. An electronic machine devised to perfor large mathematical operations on data, including decisio making. The data and the instructions are to be in som programming language. The different types of computers in u are

Analog Computers. Here the data is replaced by analogou combination of distances or voltages. For example in slide ru numbers are converted into distances.

Digital Computers. Here the numbers and data are opera ed upon after converting them into binary digits (bits).

Hybrid Computers. A combination of the analog compute and digital computers having advantages of both.

Computer Programming. The process of planning th sequence of steps of instructions to be followed by the compute

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Date 16-6-86

in solving a problem. The sequence is represented in the form of flow chart. The instructions supplied by the programmer in accordance with the flow chart are represented in the form of *coding*.

Concave, Convex Curve. A curve is said to be concave if it is curved inwards, *i.e.*, curved towards the side nearer the centre of curvature, and convex if it is curved outwards, *i.e.*, curved away from the side nearer the centre of curvature. Actually, a curve is said to be concave or convex at a point with respect to a given line if the portion of the curve in the neighbourhood of the given point lies within or outside the acute angle between the tangent to the curve at the given point and the line of reference.

Concave, Convex Polygon. A polygon is *convex* if it lies fully on one side of its any side (extended). This happens if none of its interior angle is greater than 180°, *i.e.*, π radians. If any interior angle of the polygon is greater than 180° the polygon is called *concave*.

Concavity, Convexity. The state of a curve or polygon being concave or convex.

Concentric Circles, Spheres. Circles, spheres having same centre.

Concurrent. The lines or curves having a common point.

Concyclic Points. Points that lie on the circumference of the same circle.

Condition. A proposition A which is required to be true so that another proposition B be true. If A is *necessary condition* then B could not be true without A. If A is a *sufficient condition*, then whenever A is true B is also true. For example, for a function to be differentiable the necessary condition is that the function be continuous, but continuity is not a sufficient condition for the function to be differentiable. See *Algebra of Proposition*.

Conditional. See *Algebra of Propositions*.

Conditional Probability. See *Probability.*

Cone. A solid whose base is a plane closed curve also known as *Guiding curve* and a point outside the plane, called *vertex.* A line segment joining the vertex to a point on the plane curve, generates the curved surface of the cone when the point moves on the guiding curve, the line is called the *generator* and the guiding curve is sometimes called the *directrix* of the cone. Any line segment from the vertex to the directrix is also called as *element* of the cone. If the directrix is a circle the cone is a *circular* cone and the line joining the vertex to this centre is the *axis* of the cone. If the axis is perpendicular to the base, the cone is a *right circular cone*, otherwise *oblique* cone.

The volume of right circular cone is to $1/3\ \pi r^2 h$ and the area of the curved surface is $\pi r l$, where r is the radius of the base, h the height, and l the slant height, *i.e.*, the length of an element.

Configuration. An arrangement of points, lines, curves, and surfaces which represent a geometric figure.

Confocal Conics. Conic sections having the same foci. For example, the equation $\frac{x^2}{a^2-k}+\frac{y^2}{b^2-k}=1$ represents a family of confocal ellipses if $a^2>b^2>k$ and a family of confocal hyperbolas if $a^2>k>b^2$ and the equation $y^2=4a(x-a)$ represents confocal parabolas with origin as focus and latus rectum $4a$ for varying values of a.

Conformable Matrices. See *Matrices.*

Conformal mapping. A mapping (transformation) from one geometry to another such that the angle of intersection between two tangents to the first curve remains the same as the angle of intersection between the corresponding tangents of the mapped curves.

Congruence. The state of being congruent. See *Congruent.*

Congruent. Two figures which are identical in shape and size. The figures may be in a plane or in space. The two plane figures will be congruent if one figure can be brought by rigid motion to other figure so as to fit into the area occupied by each other, coinciding exactly. For example, two circles are congruent if they have same radius. Two triangles are congruent if

(*i*) Two sides and included angle of one triangle are equal to the corresponding two sides and included angle of the other (SAS rule).

(*ii*) Two angles and the included side of one triangle are equal to the corresponding two angles and included side of the other (ASA rule).

(*iii*) Three sides of one are equal to the corresponding three sides of the other (SSS rule)

Two three dimensional figures are congruent if one can be made to coincide with the other in space by rigid motion.

Congruent Numbers. The integers p and q are said to be congruent (modulo m) if the difference $|p-q|$ is exactly divisible by the integer m. This is written as $p \equiv q \pmod{m}$. For example, $15 \equiv 3 \pmod{6}$ and $25 \equiv 13 \pmod{6}$. See *Modulo.*

Conic. (Conic Section). A plane curve defined as the locus of a point which moves in such a way that the ratio of its distances from a fixed point (*focus*) and a fixed line (*directrix*) is constant. This constant ratio is called *eccentricity* of the conic. These conics are of three types. The conic with eccencity equal to 1, are called *parabolas*; eccentricity less than 1 *ellipse*; eccentricity greater than 1 *hyperbola.* While a parabola has one focus and one directrix, the ellipse and hyperbola have two foci and two directrices each. See *Parabola*, *Ellipse* and *Hyperbola.*

Conical Surface. In the definition of *conc* if the generator is extended beyond the vertex and the directrix, then the surface generated by this generator is called the conical surface. This consists of *twin nappes.* If the directrix is the circumference of a circle and the axis of the surface is at right angles to the

plane of the circle, the surface is called *right circular conical surface*. Plane sections of such a surface are known as *conic sections*.

Conjecture. A surmisé for which proof has been possible. It is developed from a set of observations for which exception have not been found and hence taken to be valid. Conjecture is usually used in contrast with *theorem* which has rigorous proof.

Conjugate Axes of Hyperbolid. See *Hyperboloid.*

Conjugate Axis of Hyperbola. See *Hyperbola.*

Conjugate Complex Numbers. Two complex numbers of the form $x+iy$ and $x-iy$ with product x^2+y^2. If $z=x+iy$ then the complex conjugate of z is $\bar{z}=x-iy$.

Conjugate Diameters. Pairs of diameters $y=m_1x$, $y=m_2x$ of a ellipse or hyperbola with a and b as semi-major axis and semi-minor axis of an ellipse or semi transverse axis and semi-conjugate axis of an hyperbola, such that $m_1m_2=-b^2/a^2$. In case of an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ if there are two points on it with eccentric angles as ϕ_1 and ϕ_2 then the pair of diameters through these points will be conjugate diameters if

$$\phi_1\sim\phi_2=\pi/2.$$

Conjugate Hyperbolas. Two hyperbolas for which the transverse and conjugate axis of one are the conjugate and transverse axes of the other. The hyperbolas with equations

$$\frac{x^2}{a^2}-\frac{x^2}{b^2}=1 \text{ and } \frac{y^2}{b^2}-\frac{x^2}{a^2}=1$$

are conjugate of each other.

Conjunction. See *Algebra of Propositions.*

Connectives. See *Algebra of Propositions.*

Consequent. See *Antecedent, Consequent* in logic.

Conservation of Energy. The law or principle in classical mechanics that the total energy in the universe remains constant. This implies to all forms of energy, *e.g.*, heat, light, sound. If the system refers to particles in motion then we say that the sum of the *kinetic energy* and the *potential energy* remains the same. This is sometimes referred to as *principle of energy* or *energy equation* also. These days this principle also includes the concept of mass. In that case this is called the *principle of mass and energy*, or *mass-energy equation.*

Conservation of Mass. See *Conservation of Energy.*

Conservation of Momentum. This law states that the sum of the vector quantities m_1u_1, the momentum for one body and $m_2\ u_2$ the momentum for another body remains unaltered after impact. Thus if v_1, v_2 are the velocities of the two bodies after impact, then we may write

$$m_1\ u_1+m_2\ u_2=m_1\ v_1+m_2\ v_2$$

If the bodies remain in contact after the impact then

$$(m_1+m_2)\ v_3=m_1\ u_1+m_2\ u_2.$$

where v_3 is the common velocity after the impact.

Conservative Field. A field in which the work done in moving a particle from one point to another is independent of the path traversed by the particle.

Conservative Force(s). If a system of particle moves from one point to other under the action of a force or forces such that the energy transfer (work done) does not depend upon the path taken between the two points.

The system of forces in such case is said to form a *conservative field.* Force of gravitation and force of attraction are examples of conservative forces.

Consistency. See *Order Relations.*

Consistent Equations. A set of simultaneous equations which are satisfied by at least one set of values of the unknowns. For example, the equations $x+2y=5$, $2x-3y+4=0$ have the solution $x=1$, $y=2$ while the equations $x+y=5$, $2x+2y=7$ are inconsistent.

Constant. A quantity which does not change its value under specified conditions. These are of two types.

Arbitrary Constant. A constant to which one may assign different values according to specified conditions. For example, constant of integration, parameter. In the indefinite integral we add a constant of integration which is of arbitrary nature whose actual value depends on the limits chosen. See *Indefinite Integral.*

Absolute Constant. A constant value is independent of any conditions. For example, numbers.

Constant Different Series. Syn. *Arithmetic Series.*

Constant of Gravitation. See *Gravitational Constant.*

Constant of Integration. The arbitrary constant added to the expression obtained from indefinite integral.

Constant Ratio Series. Syn. *Geometric Series.*

Constant Term. In any algebraic expression the term not containing variables ; *e.g.*, in $y=ax^2+bx+c$, c is the constant term.

Continued Fraction. An integer and a fraction, the denominator of which is also an integer and a fraction, etc. Its general form is

$$a+\cfrac{b}{c+\cfrac{d}{e+\cfrac{f}{g+\text{ etc.}}}}$$

which is also written in more convinient form as

$$a+\frac{b}{c}+\frac{d}{e}+\frac{e}{f}+\text{ etc.}$$

The number of terms here may be finite (terminating) or infinite (non-terminating). The transcedental numbers such as π can be expressed in the continued fraction form.

Continued Product. The product of a finite or infinite number of terms, denoted as $\prod_{n=1}^{m} U_n$ or $\prod_{n=1}^{\infty} U_n$ respectively.

The transcedental number π can be expressed in the form of a continued product with infinite number of terms.

Continuous Function. A function $f(x)$ is said to be continuous at a point $x=a$ if $\lim_{x \to a} f(x)$ exists and is equal to $f(a)$. If this is not satisfied, we say that the function is discontinuous at $x=a$, or has discontinuity there. $f(x)$ is said to be continuous in the interval from $x=a$ to $x=b$ if it is continous at all the points of this interval. For example, $\sin \theta$ $\cos \theta$ are continuous at every point while $\tan \theta$ is discontinuous at $\theta = \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}$, etc.

Continum. Consider a line drawn infinitely on both side and having an origin. Now every point on the line corresponds to some real number and to every number there corresponds some point on the line. The points on the line are called continum of one dimension. Similarly, time is considered as a continum.

Contours. Lines or curves or a map drawn through the points of same elevation, *i.e.*, height.

Convergence. The property of an infinite sequence or series being convergent.

Convergent Sequence. A sequence $\{x_n\}$ is said to be convergent to a quantity x if $\lim_{n \to \infty} (x_n) = x$.

For example, the sequence $\left\{1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \cdots\right\}$ converges to zero.

Convergent Series. Let S_n denote the sum of first n terms of the infinite series Σu_n, then if the sequence $\{S_n\}$ tends to S as $n \to \infty$ the series Σu_n is said to converge to a sum S.

Compare *Divergent Series.* See *Ratio Test, Comparasion Test.*

Converse. A statement expressed in opposite way. For example, the converse of the statement that if two chords of a circle are equidistant from the centre, the chords are equal, is, that if two chords of a circle are equal these are equidistant from the centre Here the converse is true, but the converse of the statement that if the series Σu_n is convergent, $u_n \to 0$ as $n \to \infty$, is not true, that is if $u_n \to 0$ as $n \to \infty$ the series Σu_n may or may not converge.

Conversion Graph. A graph exhibiting a relationship between two variable quantities of physical importance, so that if a value of one quantity is given, the corresponding value of the other quantity can be read directly from the graph. For example, the conversion graph between the atmospheric pressure and the height of the place.

Convex. See *Concave, Convex.*

Coordinate Geometry. See *Analytic Geomtery,*

Coordinates of a point. An ordered set of numbers which defines the position of a point in a plane or space. See *Analytic Geometry, Cartesian, Polar, Spherical* and *Cylindrical Coordinates.*

Coplanar. Lying in the same plane. Lines which lie in one plane are called *Coplanar lines,* points which lie in one plane are called *coplanar points.* Similarly, about vector quantities, if forces lie in one plane these are called *coplanar forces.*

Corollary. A statement appended to a theorem or lemma. which can be immediately deduced from the proof already given.

Correct to n places of decimal. A number with decimal fraction expressed with n decimal places where the nth decimal digit has a rounded off value. Thus, if the $(n+1)$th place digit is greater than or equal to 5 the nth place digit is increased by one. For example, $\pi = 3.1415926\ldots$ is 3.1416 correct to four places of decimal. See *Approximation.*

Correlation Coefficient. See *Coefficient of Correlation.*

Correspondence. Syn. *Mapping or Function.* Relationship between some or all elements of one set and some or all elements of another set. See *Mapping. Function.*

Corresponding Angles. See *Transversal.*

Cosecant (*Abr. cosec*). See *Trigonometric Functions.*

Cosecant, Hyperbolic. (*Abr. cosech*). See *Hyperbolic Functions.*

Cosine (*Abr. cos*). See *Trigonometric Functions.*

Cosine Formula. In any triangle ABC if a, b, c are the lengths of its sides we have

$$\cos A = \frac{b^2+c^2-a^2}{2bc}, \quad \cos B = \frac{c^2+a^2-b^2}{2ac},$$

$$\cos C = \frac{a^2+b^2-c^2}{2ab}.$$

Cosine, Hyperbolic. (*Abr. cosh*). See *Hyperbolic Functions.*

Cotangent. (*Abr. cot*). See *Trigonometric Functions.*

Cotangent, Hyperbolic. (*Abr. coth*). See *Hyperbolic Functions.*

Countable Set. Syn. *Denumerable Set.* Any set whose elements have one-one correspondence with the elements of the set of positive integers.

Counter clockwise. See *Clockwise, Anti-clockwise.*

Couple. Two parallel forces equal in magnitude and opposite in direction form a couple. The couple when acted upon a body causes the body to turn about an axis perpendicular to the plane of forces. If F denotes the magnitude of each of the forces and d the arm of the couple then the *moment* the couple (*torque*) about any point in the plane is equal to Fd.

Covariance. A measure of association between two variables. If x and y are any two variables with $\bar{x}$ and $\bar{y}$ their means, n is the number of pairs of values of x and y, then the covariance between the variables is defined as

$$\Sigma[(x_i-\bar{x})(y_i-\bar{y})]/n.$$

C.P.U. Central Processing Unit also known as central processor. See also *Computer*.

Crammer's Rule. A rule for solving simultaneous equations by determinant method. For example, if we have three simultaneous algebraic equations.

$$a_1x+b_1y+c_1z=d_1,\ a_2x+b_2y+c_2z=d_2,\ a_3x+b_3y+c_3z=d_3.$$

and if $D=\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_1 \\ a_3 & b_3 & c_3 \end{vmatrix}\neq 0$, $D_1=\begin{vmatrix} b_1 & c_1 & d_1 \\ b_2 & c_2 & d_2 \\ b_3 & c_3 & d_3 \end{vmatrix}$,

$D_2=\begin{vmatrix} a_1 & c_1 & d_1 \\ a_2 & c_2 & d_2 \\ a_3 & c_3 & d_3 \end{vmatrix}$, $D_3=\begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix}$

then $x=\frac{D_1}{D}$, $y=\frac{-D_2}{D}$, $z=\frac{D_3}{D}$ $(D\neq 0)$. This is known as Crammer's rule. It can also be applied to a general case of n algebraic equations with n unknowns.

Critical Point. Any point on the curve $y=f(x)$ at which the gradient $\frac{dy}{dx}$ is either zero or infinite, *i.e.*, at which the tangant to the curve is parallel to the either of the coordinate axes.

Cross Multiplication. Replacing the equality of the ratios, *i.e.*, $\frac{a}{b}=\frac{c}{d}$ by the equivalent statement

$$a\,.\,d=b\,.\,c.$$

Cross Product. Syn. *Vector Product.*

Cross Section. A geometric figure obtained by cutting a solid figure by a plane or a plane figure by a straight line. For

ample, the cross section of a sphere by a plane is a cle.

Cube. (1) A quantity raised to the power three, *e.g.*, x^3 ich is equal to $x.x.x$. (2) In geometry it is a regular solid ure having six square faces, 8 vertices, and 12 edges. The lume of a cube is a^3 if a is the length of its side.

Cube Root, A quantity whose cube (third power) is equal a given quantity. If n is a given quantity then its cube root x if $x^3=n$ so that $x=n^{1/3}$ or $x=\sqrt[3]{n}$. For example 2 is the be root of 8.

Cubic. (1) Having the properties of cube in algebra.

(2) A figure which has the properties of cube in geometry.

Cubic Equation, An algebraic equation of degree three the form $a_0x^3+a_1x^2+a_2x+a_3=0$ where $a_0\neq 0$. It has three ots out of which at least one root has to be real. If only one ot is real the other two will be complex roots conjugate of ch other. The roots of the equation $x^3=1$ are ω and ω^2 here $1+\omega+\omega^2=0$.

Cubic Expression. An algebraic expression of degree ree. For example, $a_0x^3+a_1x^2+a_2x+a_3$, where $a_0\neq 0$.

Cubic Measuae. Measurement of volume taking 1 cm^3 or ft^3 as a basic unit.

Cubic Numbers. The numbers 1, 8, 27, 64, ... *i.e.* 1^3, , 3^3, 4^3, etc. A typical property of cubic numbers is at any cubic number n^3 can always be expressed as a sum of cosecutive odd numbers begining with $n(n-1)+1$. Thus,

$$8=3+5,$$
$$27=7+2+11,$$
$$64=13+15+17+19.$$

Cubic Parabola. A curve whose Cartesian equation is of e form $y=ax^3$ or $x=ay^3$.

Cuboid. A solid figure of box type having six rectangular ces and eight vertices.

Curl. (Symbol $\nabla\times$). A vector function obtained by perating a differential operator upon a vector point function. or example, if $F=F_1i+F_2j+F_3k$, then

$$\nabla \times \vec{F} = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_1 & F_2 & F_3 \end{vmatrix}$$

where i, j, k are the unit vectors along the three axes The curl of a vector point function is always a vector. In general, the curl of a vector point function gives the measure of angular velocity. Any motion in which the curl of velocity vector is not zero is said to be rotational, otherwise irrotational.

Curvature. Numerical measure of how much a curve is curved at a given point on it. It can be defined as the rate of change of slope of the tangent of a curve with respect to the length of the arc upto the point of the curve. For each point on a curve there is a circle called *circle of curvature* which has the same tangent and same curvature at that point. The radius of this circle is called *radius of curvature* denoted by ρ and curvature is denoted $1/\rho$. The centre of the circle of curvature is called *centre of curvature*. The radius of curvature of a circle is equal to its radius. For the curve $y=f(x)$,

$$\rho = \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{d^2y/dx^2} \qquad \left(\frac{d^2y}{dx^2} \neq 0\right)$$

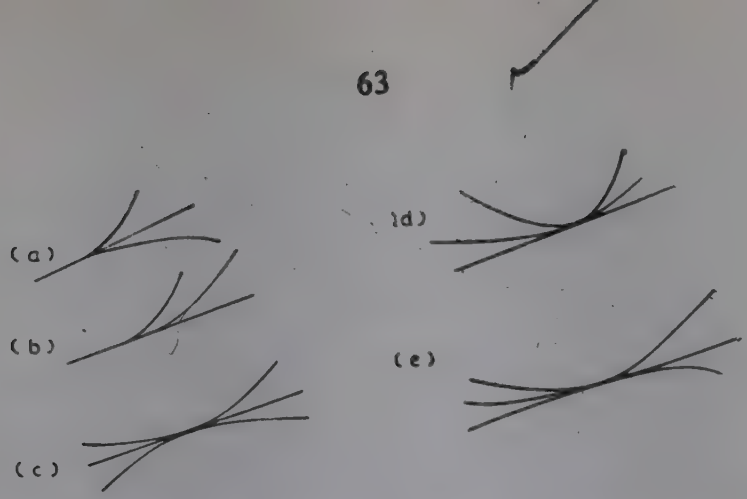
Curve. A set of points forming a one-dimensional configuration obeying some law. For example $y=ax^2$ forms a parabola.

Curve-Fitting. See *Line of Best Fit.*

Curve Tracing. The process of plotting points to give rise to the graph of a curve. It includes the use of techniques of symmetry, maxima and minima, curvature, asymptotes, etc.

Curvilinear Motion. Motion along a curve.

Cusp. A *double point* at which there are more than one tangents that are real and coincident. Cusps are of following types (*a*) single cusp of first species, (*b*) single cusp of second species, (*c*) double cusp of first species, (*d*) double cusp of second species, (*e*) double cusp of both species. See figures.



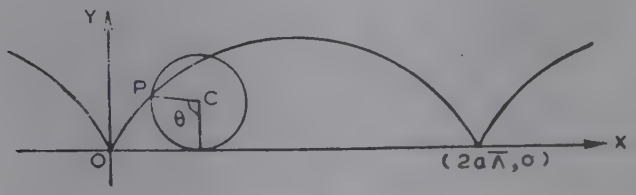
Cycle. A sequence of events that are regularly repeated, *e.g.*, revolution (rotation), vibration, oscillation, wave motion.

Cyclic Group. A group in which each element can be expressed as power of another element.

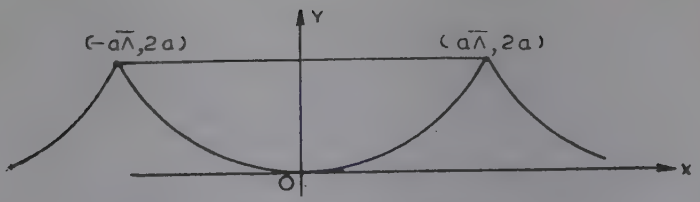
Cyclic Order. The way in which a cyclic permutation is performed. See *Permutation*.

Cyclic Polygon. A polygon with all its vertices on the circumference of a circle. All triangles are cyclic, squares and rectangles are cyclic quadrilaterals, all regular polygons are cyclic.

Cycloid. The path traced out by a point on a circle which roles on a straight line. If C is the centre of the circle with radius a and if θ is the angle of rotation at any instant, *i.e.*, angle between CP and the vertical line through C, then the parametric equation of the curve is $x=a(\theta-\sin\theta)$, $y=a(1-\cos\theta)$.



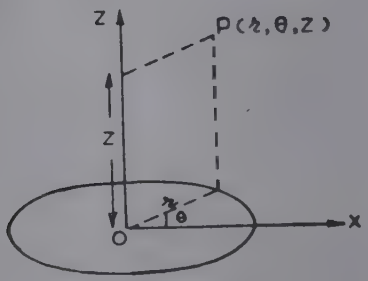
A Cusp occurs whenever the point P touches the base line. Distance between any two cusps along the base line is $2a\pi$, the circumference of the circle, and distance along the arc of the cycloid is $8a$.



The inverted cycloid has got the parametric equation as $x=a(\theta+\sin\theta)$, $y=a(1-\cos\theta)$.

Cylinder. A solid defined by a closed curve called *guiding curve* (forming a base) or *directrix*, with an identical curve parallel to it. Any line segment from a point on one curve to a corresponding point on the other curve is an *element* of the cylinder. If one such element moves parallel to itself round the base it generates the curved lateral surface. The line is called generator of the cylinder. If the base is a circle then the cylinder is a circular one. The line joining the centres of such circular bases is called *axis* of the circular cylinder. If the axis is perpendicular to the circular base the cylinder is called the right circular cylinder. The volume of a right circular cylinder with radius of its base as r and its height h is $\pi r^2 h$ and the curved lateral surface is $2\pi rh$.

Cylinderical Coordinates. Coordinates of a point in space as (r, θ, z) where r is its horizontal distance from fixed vertical axis called z-axis, θ is the angle, the horizontal radius made with an axis in the horizontal plane called X-axis and z is the height of the point above the fixed horizontal plane called X-Y-plane.



For a point $P(r, \theta, z)$ the corresponding rectangular cartenian coordinates (x, y, z) are $x=r\cos\theta$, $y=r\sin\theta$ and $z=z$.

The coordinates are described as cylindrical since the point describes in cylindrical surface when r is constant.

D

D' Alemberts' Ratio Test. The generalized ratio test for determining whether a given infinite series is convergent or divergent. According to this test the series $\sum_{n=1}^{\infty} u_n$ is convergent or divergent if, from and after certain term, the absolute value of the ratio $\frac{u_{n+1}}{u_n}$ is respectively less than or greater than some fixed number less than unity. In other words if $\text{Lt}_{n\to\infty} \frac{u_{n+1}}{u_n} = l$, the series is convergent or divergent according as l is less than or greater than 1. See *Convergence, Divergence of Infinite Series.*

Data. Plural of the Latin word *Datum* which means 'that which is given'. The information that describes an object, idea, condition etc., that provides basis for establishing propositions, hypotheses, inferences, proofs etc. For computations, data is regarded as facts on which a program runs. Data is accepted and processed in a computer in binary form.

Data Processing. Set of operations performed upon data to extract information or to achieve some form of order. The data is processed by a computer.

Debug. To locate and detect errors (*bugs*) in a program run on a computer.

Deca. A prefix standing for 10. For example,

1 decagram = 10 grams.

Decagon. A plane figure with ten sides.

Decahedron. A solid figure with ten faces.

Deci. A prefix standing for 10^{-1}. For example,

$$1 \text{ decimetre} = \frac{1}{10} \text{ metre.}$$

Decimal. System based on base ten. A decimal fraction is a rational number written as units, tenths, hundredth, thousandths, and so on. For example, 1/8 is ·125 in decimal notation. A decimal fraction having finite number of digits is called a *finite* decimal or *terminating* decimals, while a decimal fraction having infinite number of digits is called *infinite* and *non-terminating* decimal. Some rational numbers as 5/3 (=1.666..... denoted by $1.\dot{6}$) cannot be written as exact decimal and in these some digits repeat infinitely. These are called *repeating* or *recurring* decimals. Thus, all the rational numbers can be represented either as a finite decimal or as recurring decimal. All the irrational numbers are represented by non-repeating or non-recurring decimals.

Deduction. Conclusion derived from data by a process of reasoning.

Definite Integral (Riemann Integral). The definite integral $\int_a^b f(x)\,dx$ of a function $f(x)$ is the area bounded by the curve $y=f(x)$ and the two ordinates at $x=a$ and $x=b$. The area above the x-axis is taken as positive while that below the x-axis as negative. If $g(x)$ stands for the indefinite integral $\int f(x)\,dx$, then the definite integral $\int_a^b f(x)\,dx = g(b) - g(a)$.

Here a and b are called the *lower and upper limits of integration*

Definition. The description of a new term or idea by using nly ideas accepted but not yet finalized, called axioms or ostulates and the already defined symbols and operations.

Deformation. The geometric changes in the shape and ze of a body under the action of certain forces. The defor- nations are temporary in case of elastic bodies and are ermanent in case of plastic bodies. When deformation takes lace in a body stresses develop. See *Stress.*

Degenerate Conic. The conic sections which are neither lliptic, parabolic, nor hyperbolic.

Degree. (1) A unit of plane angle equal to one three undred and sixtieth part of a revolution.

(2) The exponent or index of a variable. For example, $5x^6$ is of sixth degree and $2x^2+3x+7$ is of second degree. f there are several terms then degree is sum of exponents. For example $3x^2y^3$ is of degree five. (3) The degree of a differential equation is the highest power of the highest order derivative in the differential equation. (4) Unit of temperature. See *Celsius, Farenheit.*

De-Moivres' Theorem. According to this theorem for all n, positive or negative, integer or fraction the value or one of the values of $(\cos\theta+i\sin\theta)^n$ is $\cos n\theta+i\sin n\theta$. It is used to calculate the value of a complex number raised to some power.

De Morgan's Laws. If A, B, C, ... are subsets of a set R and a dash denotes the complement of a set, then

$$(A\cup B\cup C\ldots)'=A'\cap B'\cap C'\cap\cdots$$

$$(A\cap B\cap C\ldots)'=A'\cup B'\cup C'\cup\cdots$$

Denominator. The lower part of a fraction. For example, in a fraction $\frac{5}{3}$, 3 is the denominator.

Denumerable Set. Syn. *Countable Set.*

Dependent, Independent Variable. See *Variable.*

Derivative. Syn. *Differential Coefficient. Result o differentiation.* If y is a function of x, given by $y=f(x)$, th first order derivative of y is $\frac{dy}{dx}$ $[\equiv f'(x)]$. If $y=u.v$ wher u and v are functions of x then

$$\frac{dy}{dx}=\frac{du}{dx}\,.\,v+u\,.\,\frac{dv}{dx}$$

and if $y=\frac{u}{v}$ then $\frac{dy}{dx}=\left(v\,\frac{du}{dx}-u\,\frac{dv}{dx}\right)\Big/v^2$.

Higher order derivatives can also be defined, so that second order derivative is $\frac{d}{dx}\left(\frac{dy}{dx}\right)$ denoted by $\frac{d^2y}{dx^2}$ or by y'' sometimes. The n^{th} order derivative of y is denoted by $\frac{d^ny}{dx^n}$ or by $y^{(n)}$.

Descartes' Rule of Signs. See *Rule of Signs and Roots.*

Descending Order. See *Ascending Order.*

Determinant. A square array of quantities known as *elements.* If there are n^2 elements, there must be n rows and n columns, and n will be called the *order of the determinant.*

For example, the *second order determinant,* is

$$\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}=a_1\,b_2-a_2\,b_1.$$

Third order determinant is

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

$$=a_1b_2c_3-a_1b_3c_2+a_3c_2b_1-b_1a_2c_3+c_1a_2b_3-c_1a_3b_2$$

$$=a_1\begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix}-a_2\begin{vmatrix} b_1 & c_1 \\ b_3 & c_3 \end{vmatrix}+a_3\begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix}$$

$$= a_1 A_1 - a_2 A_2 + a_3 A_3$$

where

$$A_1 = \begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix}, \quad A_2 = \begin{vmatrix} b_1 & c_1 \\ b_3 & c_3 \end{vmatrix}, \quad A_3 = \begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix}$$

where A_1, A_2, A_3 are *minors* of the determinant.

Determinants are expanded by the *rule of Sarrus.*

Cramer's Rule uses these notations for solving simultaneous equations.

Deviation. Variation from a standard of reference. See *Mean Deviation, Standard Deviation.*

Diagonal. A line segment joining any two non-adjacent vertices of a polygon.

Diagonal Matrix A square matrix having all its elements zero except those in the leading diagonal. A matrix $A[=(a_{ij})]$ is diagonal matrix if $a_{ij}=0$ for $i \neq j$.

Diameter. A chord through the centre. For example, diameter of a circle, of a conic, of a sphere.

Difference. The remainder obtained on subtracting one quantity from another. Two quantities are equal if their difference is zero.

Difference of Two Sets. The set of all elements belonging to a set A but not belonging to a set B. It is written as $A-B$.

Difference of Two Vectors. See *Composition of Vectors.*

Difference Operator. See *Finite Differences.*

Differential. An infinitesimal change in an independent variable or in a dependent variable as a result of small change in independent variable. dx and dy are differentials in $\frac{dy}{dx}$, the differential coefficient of y w.r.t. x.

Differential Calculus. See *Calculus.*

Differential Coefficient. Syn. *Derivative.* For a function $y=f(x)$ the differential coefficient of y w.r.t. x is defined as

$$\frac{dy}{dx} = \lim_{\delta x \to 0} \frac{f(x+\delta x)-f(x)}{\delta x} = \lim_{\delta x \to 0} \frac{\delta y}{\delta x}.$$

It is also denoted by y' or Dy where $D \equiv \frac{d}{dx}$.

Differential Equation. Any equation which contains derivatives. For example, $\frac{dy}{dx}+4y+6x-17=0$, is a differential equation. The order of the highest order derivative is is called the *order of the differential equation* and the degree of the differential equation is the degree of the highest order derivative. The above differential equation is of first order and of first degree. See *Degree* also.

Differential Geometry. The branch of mathematics in which calculus is applied to the theory of waves and surfaces in a three demensional space.

Differential Operator. Operator used in differentiation. It is usually denoted by D where $D \equiv \frac{d}{dx}$.

Differentiation. The process of finding the derivative of a function. It stands for finding the rate of change of one variable with respect to another. For example, if a particle has traversed a distance s at any instant t and a distance $s+\delta s$ at any time $t+\delta t$, the velocity of the particle at a time t is given by $\frac{ds}{dt} = lim \frac{\delta s}{\delta t}$. Similarly, $\frac{d^2s}{dt^2}$ stands for the acceleration of the particle of any time t, which is the rate of change of velocity.

Digit. A symbol associated with primitive counting. One or more number of digits form numbers. For example, in a number 279 there are three digits, namely, 2, 7, and 9. The decimal number system has ten digits (0—9), whereas binary system has only two digits 0 and 1.

Digital. Anything using digits. Digital devices generally work by some sort of counting process. Digital computers are very common and highly accurate.

Dihedral. A figure formed by two intersecting planes. The straight line in which two planes intersect is called the *edge* of dihedral and the two planes are called its *faces*. The dihedral angle is the angle between its two faces, which is measured by the angle between the lines of cross-section of the two planes and a plane perpendicular to the above line of cross-section.

Dimension. (1) The least number of coordinates needed to represent a point in a plane or in space. A point in a plane requires two coordinates hence a plane figure will be two dimensional. Similarly, a solid is three dimensional. Some abstract spaces are taken as n-dimensional. (2) The size of a plane figure or a solid. For instance, the dimension of a circle is its radius, that of a rectangle are its length and breadth, and of a rectangular parallelopiped are its length, breadth and height. (3) If L is the dimension of length, M that of mass and T that of time then the dimension of volume is $[L]^3$, dimension of velocity is $[L]\,[T]^{-1}$, that of force is $[M]\,[L]\,[T]^{-2}$, that of stress is $[M]\,[L]^{-1}\,[T]^{-2}$. (4) Dimensions of a matrix are its number of rows and those of its columns.

Direct Variation. A variation of the form $y=kx$ where k is constant of proportionality. We say y varies directly with x.

Directed. Any line affixed with a direction is called *directed line* which is positive in one direction and negative in the opposite direction. A part of a directed line is called *Directed line segment*. For example, $\overrightarrow{AB}$ which is same as $-\overrightarrow{BA}$. Directed line segments are used to represent vectors.

Direction. Through a point A, say, pass infinite number of straight lines, each defining a direction through A. Further only one of these pass through a fixed point B. This defines the direction $\overrightarrow{AB}$ or $-\overrightarrow{BA}$.

Direction Angles. The angles made by a line with the positive directions of x-axis, y-axis, z-axis.

Direction Cosines. *Cosines of the direction angles.* If a line makes angles α, β, and γ with the x-axis, y-axis, z-axis respectively then direction cosines of the line are $\cos\alpha$, $\cos\beta$, $\cos\gamma$. These are generally denoted by letters l, m, n, so that $l=\cos\alpha$, $m=\cos\beta$, $n=\cos\gamma$.

Direction Ratio. Any three quantities a, b, c in constant proportion to l, m, n the direction cosines of a line, *i.e.*

$$\frac{a}{l}=\frac{b}{m}=\frac{c}{n}=k \text{ say.}$$

Directional Derivative. Rate of change of a function with respect to distance s in a specified direction or along a specified curve. If α, β, γ are the angles made by the specified direction or the tangent to the specified curve at the point $P(x, y, z)$ with the three coordinate axes then the directional derivative of a function $f(x, y, z)$ is

$$\frac{df}{ds}=\frac{\partial f}{\partial x}\cos\alpha+\frac{\partial f}{\partial y}\cos\beta+\frac{\partial f}{\partial z}\cos\gamma.$$

The directional derivative is maximum in direction of the normal to the surface and is called the gradient of f denoted by ∇f. See *Gradient.*

Director Circle. The locus of the point of intersection of a pair of tangents to a central conic which are at right angles to each other. For the central conic

$$ax^2+2hxy+by^2+2gx+2fy+c=0,$$

the equation of the director circle is

$$(ab-h^2)(x^2+y^2)-2(hf-bg)x-2(gh-af)y+c(a+b)-(f^2+g^2)=0$$

Directrix. (1) A fixed straight line connected with a conic ection, whose distance from a variable point on the conic is n constant ratio to the distance of the variable point from a fixed point called focus. (2) Syn. *Guiding Curve.* A plane curve which defines the base of a cone or cylinder.

Disconitinuity. A function which is not continuous at a point is said to posses discontinuity at that point. See *Continuous Functions.*

Discontitinuous. Function having disconitnuity at a point s said to be discontinuous at that point. See *Continuous Function* and *Disconitinuity.*

Discrete. Property of set of numbers or events being such hat there are no intermediate elements in between any two elements. For example, set of integers is discrete while set of rational numbers is not discrete.

Discriminant. The expression b^2-4ac in a quadratic equation $ax^2+bx+c=0$, which decides the nature of roots. The roots are real, complex, or zero according as the discriminant is positive, negative, or zero.

Disjoint. Two sets are called disjoint if they donot have any common elements. Two events are said to be disjoint if these are independent.

Disjunction. See *Algebra of Propositions.*

Disk. A device used in computers for storing information for afterwards use. See *Magnetic Disk.*

Dispersion. A term in statistics used as a measure of variation of data from some central tendancy, say mean. It is usually measured by the *standard deviation.* See *Standard Deviation.*

Displacement. Vector quantity for distance between two positions of a moving object. Since displacement is a vector it has magnitude as well as direction. Displacement may be rectilinear, angular, or both.

Distance. A measure of length of path between two points. It is usually denoted by s. Distance between two points (x_1, y_1, z_1) and (x_2, y_2, z_2) is

$$\sqrt{(x_1-x_2)^2+(y_1-y_2)^2+(z_2-z_1)^2}$$

Distribution. A term in statistics describing the set of values of a variable and the frequencies of occurrence of each value. Syn. *Frequency Distribution.*

Distribution Function. The function $f(x)$ for a random variable x, which is equal to the probability of occurrence of that value of x.

Distributive Property. A law which connects the operations of multiplication and division. In general, according to this law or property of multiplication over addition $a\times(b+c)=a\times b+a\times c$. In Boolean algebra, if it refers to addition over multiplication, $a+(b\times c)=(a+b)\times(a+c)$ whereas in set-theory $A\cap(B\cup C)=(A\cap B)\cup(A\cap C)$.

Divergence (1) (Div.). Symbol ∇. A vector operator which when operated on a vector point function $\vec{F}(x, y, z)$ gives a scalar quantity as follows. If $\vec{F}=F_1i+F_2j+F_3k$ then

$$\nabla \cdot F=i \cdot \frac{\partial F}{\partial x}+j \cdot \frac{\partial F}{\partial y}+k \cdot \frac{\partial F}{\partial z}=\frac{\partial F_1}{\partial x}+\frac{\partial F_2}{\partial y}+\frac{\partial F_3}{\partial z}.$$

(2) The phenomenon of an infinite series being *divergent*. Opposite to *convergence.*

Dividend. A term in division. The number which is divided by another number (divisor) to give a *quotient* and a *remanider.* For example, in $\frac{22}{7}$, 22 is dividend, 7 is divisor, 1 is remainder.

Dividendo. See *Proportion.*

Division. An operation, inverse of multiplication. It is a binary operation of finding the quotient of two quantities. See *Multiplication.*

Divisor. See *Dividend.*

Documentation. Complete instructions alongwith all comments necessary to describe fully the program to be executed on a computer.

Domain. See *Function* or *Mapping.*

Dot Product. Syn. *Scalar Product.*

Double Integral. See *Multiple Integrals.*

Double Point. A second order *multiple point.* A point common to two branches of a curve, at which there are two tangents. There are several types of double points. (*a*) The curve crosses over it self forming a loop, and has two distinct tangents, it is called *node* ; (*b*) the curve doubles back on itself and has only one tangent, it is called *cusp* ; (*c*) two arcs of the curve touch each other and have common tangent but unlike a cusp, the arcs continue beyond the double point and make four branches of the curve, it is called *tacnode* ; (*d*) a point which satisfies the equation of the curve but does not lie on the main arc of the curve, is known as *isolated double point.*

Double Root. Syn. *Repeated Root.*

Dynamics. The branch of mathematics which studies the motion of objects under the action of applied forces. This involves the relation between force and motion and also the description of the motion When the bodies under motion are neither rigid nor elastic, *e.g.*, fluids, we call this branch *Hydrodynamics.*

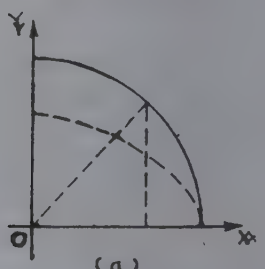
Dyne. A unit of force in C.G.S. system and is defined as a force which produces an acceleration of one centimetre per second in a mass of one gram.

E

e. *Euler's constant.* It is the natural base of logarithm and is given by

$$e = \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = \lim_{n\to 0}(1+n)^{1/n}.$$

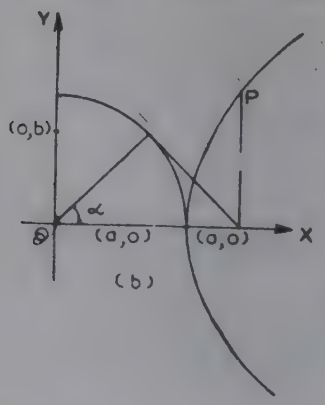
Eccentric Angle for Ellipse. Let the ellipse have the equation $x^2/a^2+y^2/b^2=1$, with a and b as semi-major and semi-minor axes and centre of ellipse at the origin. Draw an auxiliary circle of ellipse, which is the circle with centre at the centre of the ellipse and radius equal to the semi-major axis a. If $P(x, y)$ is any point on the ellipse the eccentric angle is the angle between OX and OP' when P' is the point on the auxiliary circle with the same abscissa as of P. If ϕ is eccentric angle of the point $P(x, y)$ then the *parametric* equations of the ellipse can be written as $x=a\cos\phi$, $y=b\sin\phi$, ϕ being parameter.



(a)

Eccentric Angle for Hyperbola. Let the hyperbola have equation $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$, with a and b as semi-transverse and semi-conjugate axes and centre at the origin. Draw the auxiliary circle of the hyperbola, which is the circle with centre at the centre of the hyperbola and radius equal to semi-transverse axis a. If $P(x, y)$ is any point on the hyperbola, the eccentric angle at the point P is the angle between OX and OP' where P' is the point on the auxiliary circle with the same abscissa as

of P If θ is the eccentric angle of the point $P(x, y)$ then the parametric equations of the hyperbola can be written as $x = a \sec \theta$, $y = b \tan \theta$, θ being the *parameter.*



Eccentricity. The ratio of the distances of any point on a conic from the fixed point (focus) and the fixed line (directrix). It is a constant and is denoted by e. The conic is parabola, ellipse, or hyperbola according as e is equal to, less than, or greater than unity.

Edge. A straight line where two faces of a solid meet. For example, a tetrahedron has six edges and a cube has eight edges.

Elasticity. The property of a body which resists deformation and brings the body to its original shape and size when the external forces responsible for deformation are removed. The theory of elasticity is based on the Hooke's law which establishes relationship between stress and strain. The ratios of stress to strain are known as *Young's modulus* in longitudinal stress, *rigidity modulus* in shear stress and *bulk modulus* in volume stress.

Electromagnetic Unit (E.M.U.). A system of electrical units based on the concept of a unit magnetic pole which repels a similar pole at a unit distance, *i.e.*, at a distance1 cm with a force of 1 dyne.

Electrostatic Unit (E S.U.). A system of electric units based on the electrostatic unit of electricity which repels a similar quantity 1 cm apart with a force of 1 dyne.

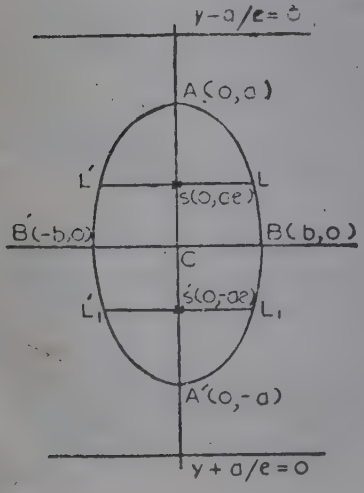
Electromotive Force (E.M.F.). Work is expended in maintaining the flow of electric current in a conductor. The energy required is drawn from some external source known as electromotive force (E.M.F.) and is measured by rate of generating heat. The rate of expending energy is a measure of E.M.F. For unit current the E.M.F. is also unity and rate of working is one erg per second.

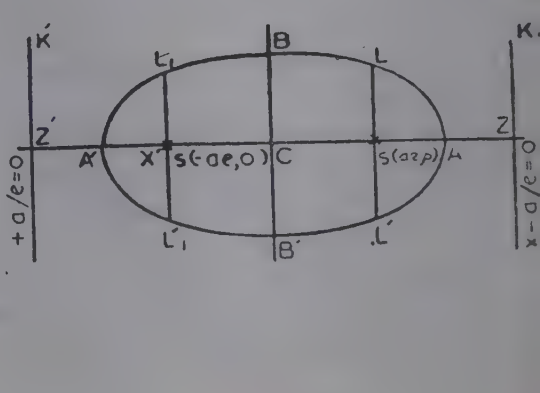
Element. (1) A member of set. For example, the number 11 is an element of a set of integers. (2) A member of an array. (3) A line segment forming part of the curved surface of a cone or a cylinder. (4) Element of length as δx, element of area as δA or $\delta x\ \delta y$, element of volume as δV or $\delta x\ \delta y\ \delta z$, as used for the purpose of summing for integration.

Elevation. Vertical distance of a point above some standard level.

Elimination. The method by which in a given set of two or more algebraic equations one of the unknowns is removed and thus the number of equations gets reduced by one.

Ellipse. A conic with eccentricity less than one. An ellipse has two foci and the line joining them is called major axis of the ellipse. The major axis cuts the ellipse at two points called vertices of the ellipse. The mid point of the major axis is called the centre of the ellipse, and the line through the centre and perpendicular to the major axis is called minor axis of the





ellipse. There are also two directrices of the ellipse. The standard equation of the ellipse is $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, when centre is at the origin and major and minor axes are along the x-axis and y-axis respectively. The coordinates of the vertices are $(\pm a, 0)$ and those of foci are $(\pm ae, 0)$, the equations of directrices are $x=\pm a/e$. The ellipse cuts the minor axis at points $(0, \pm b)$, a and b are semi-major and semi-minor axes of the ellipse which is symmetrical about both the co-ordinate axes. The chords of the ellipse through foci and parallel to minor axes are called latus recta with length $2b^2/a$.

Ellipsoid. A solid whose plane cross sections parallel to the three co-ordinate axes are ellipses It has three axes of symmetry. The equation $x^2/a^2+y^2/b^2+z^2/c^2=1$ represents the standard cartesian equation of ellipsoid with $2a$, $2b$, $2c$ as lengths of its three axes. If $b=c$ the ellipsoid is of *prolate* shape and is obtained by the revolution of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ about its major axis. If $a=b$, the ellipsoid is of *oblate* shape and is obtained by the revolution of the ellipse $\frac{x^2}{a^2}+\frac{z^2}{c^2}=1$ about its minor axis. If $a=b=c$ the ellipsoid is the sphere $x^2+y^2+z^2=a^2$.

Emperical. Derivable and demonstrable only from experiments or observations.

Emply—Set. Syn. *Null set.* A set with no element and and is denoted usually by ϕ. See *Set Theory.*

Energy. Capacity of a system to do work. Energies can be put broadly into two catagories, namely, *kinetic energy* and *potential energy.*

Entire Function. Syn. *Integral Function.*

Enumerable Set. Syn. *Countable Set, Denumerable Set.*

Enunciation. A precise statement.

Envelope. The curve which is tangent to every member of a family of lines or curves.

Epicycloid. It is the locus of a point on a circle which rolls on the outside of a fixed circle without slipping. It's parametric equation are

$$x=(a+b)\cos\theta+b\cos[(a+b)\,\theta/b]$$

$$y=(a+b)\sin\theta-b\sin[(a+b)\,\theta/b]$$

where a is the radius of the fixed circle, b the radius of the rolling circle, and θ the angle between the line of centres and the positive direction of x-axis. There is a cusp of first kind at every point of intersection of the epicycloid and the fixed circle. When $a=b$ the curve becomes a cardioid.

Equality. The property of two quantities being equal. This is usually expressed in the form of an equation. Two complex numbers will be equal if their real parts and imaginary parts are respectively equal. Thus, $a+ib=c+id$ if $a=c$ and $b=d$.

Equation. A statement of equality. If the equality is true only for certain values of the unknown quantities involved in the equation, then it is sometimes called conditional equation and the $=$ sign is used ; if the equality is true for all values of the unknown quantities involved, the equation is called an *identity* and the $\equiv$ sign is used.

Equations of Motion. The equations which describe the motion of an object. If we consider the linear motion only and acceleration to be constant, then the equations may be written as

$$v=u+ft$$

$$s=ut+\frac{1}{2}ft^2$$

or

$$s=(u+v)\,t/2$$

$$v^2=u^2+2fs$$

wher u and v denote the initial and final velocities respectively in a time interval t, s is the distance travelled in time t and f is the constant acceleration. f is replaced by $-f$ if it is retardation.

Equator. The great circle on the surface of the earth equidistant from the north pole and south poles. The equator divides the earth's surface into the northern and southern hemispheres. The plane in which equator lies is called *equitorial plane.*

Equiangular. Having equal angles.

Equiangular Spiral. Syn. *Logarithmic Spiral.* It's equation is $r=ae^{\cot \alpha}$. The angle between the radius vector of a point on the curve and the tangent at that point always remains equal to α.

Equidistant. At the same distance.

Equilateral Polygon. Polygon having all sides equal.

Equilateral Triangle. Triangle having all the three sides equal. Each of the angles is also equal to 60°.

Equilibrium. The state of a body or particle when the resultant of all the forces acting on it is zero and there is no torque acting on it. The equilibrium is *stable* or *unstable* if, when slight displacement is given to the body, the body tends to return to or move away from its original position respectively under the action of forces actting on the body.

Equivalence of Proposition. See *Algebra of Propositions.*

Equivalence Relation. A relation between elements of a set which satisfies three conditions: (*a*) *Reflexive*, *aRa*, (*b*) *Symmetric*, that is, if *aRb* then *bRa*, (*c*) *Transitive*, that is, if *aRb* and *bRc* then *aRc*, where *a*, *b*, and *c* all belong to the set. For example, congurence in geometry, equality in algebra.

Erg. Unit of work in C.G.S. system. The work done by a force of 1 dyne acting through a distance of 1 cm. 1 erg $=10^{-7}$ Joule.

Error. For any set of measures of the same quantity there is a mean which is considered to be a theoritical 'true

value' *i.e.* a unique value which exists only as a concept. Error is the difference between a particular measure and a set of measures. In statistics it can be the variation due to factors beyond control or due to sampling error *i.e.* due to the arbitrary nature of the sample chosen to represent the population.

Escribed Circle. A circle drawn externally to a polygon to touch the three consecutive sides, the first and third sides extended.

Escribed Circle of Triangle. There are three escribed circles of a triangle. The escribed circle touching the side a of a triangle ABC has radius r, equal to $\Delta/(s-a)$ where

$$s=(a+b+c)/2 \text{ and } \Delta=\sqrt{s(s-a)(s-b)(s-c)}.$$

Euclidean Algorithm. A method of finding the H.C.F. (the highest common factor) of two numbers. The smaller number divides the larger number. The remainder then divides the smaller number giving a second remainder. The second remainder then divides the first remainder to give a third remainder, and so on. The process is contined until a zero remainder achieved.

Euclidean Geometry. A system of geometry as put forward by the Greek Mathematician Euclid. He gave definitions of a point, a line, etc. and together with a number of assumptions. These were taken as basic axioms.

Euler's Constant, e. Base of natural logarithm. See e.

Euler's Theorem. (1) According to this theorem,

$$e^{ix}=\cos x+i \sin x$$

for every real x.

(2) This Euler's theorem for homogenous function states if u is a homogeneous function of degree n in $x, y, z, \ldots$, then

$$x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}+z\frac{\partial u}{\partial z}+\ldots\ldots=nu.$$

Evaluation. Finding the numerical value of some expression.

Even Function. A function $f(x)$ of a variable x is said to be even if $f(-x)=f(x)$. The graph of such a function will be symmetrical about y-axis. For example,

$$y=x^2,\ y=\cos x.$$

Even Number. An integer divisible by 2. For example,

$$0,\ \pm 2,\ \pm 4,\ \pm 6,\ \pm 8.$$

Evolute. The evolute of a given curve is the locus of the *centres of curvature* of points on the given curve.

Ex-centre. The centre of the *escribed circle.*

Expansion. The process of expressing a quantity as a sum of a series of terms. Many a times a function may be written as a convergent infinite series. The function, in that case, can be approximated to any required degree of accuracy by taking the sum of a certain number of terms from the series. The expansion of $(1+x)^n$ is

$$1+nx+\frac{n(n-1)}{2!}x^2+\frac{n(n-1)(n-2)}{3!}x^3+\ldots\ldots$$

where $-1<x<1$.

Expectation. See *Expected Value.*

Expected Value. The calculated value of a variable quantity which is most likely to occur. If a variable x can take any of the values $(x_1, x_2, \ldots\ldots, x_n)$ with corresponding probabilities $(p_1, p_2, \ldots\ldots, p_n)$ then expected value of x or expectation of x is written as

$$E(x)=p_1x_1+p_2x_2+\ldots\ldots+p_nx_n$$

On the other hand if x is a continuous variable with probability density function $p(x)$ then

$$E(x)=\int_{-\infty}^{\infty} x\,p(x)\,dx.$$

Explicit. If a variable say y, can be expressed explicitly as a function of another variable say x, as $y=f(x)$, then y is known as an explicit function of x. For example, if

$$z=3x \sin x+7 \tan^{-1}x-7$$

then z is an explicit function of x.

Exponent. Power to which a number or expression is raised. The exponent is written in the form of a subscript. For example, n and x are exponents in a^n and $(ay+d)^x$. Following are the properties of the exponents.

(*i*) multiplication : $x^a x^b=x^{a+b}$

(*ii*) division : $x^a/x^b=x^{a-b}$

(*iii*) power of power : $(x^a)^b=x^{ab}$

(*iv*) negative power : $x^{-a}=1/x^a$

(*v*) Fractorial power : $x^{a/b}=\sqrt[b]{x^a}$

Exponential Function. A quantity which varies as the power of another quantity. It is a transcedental function with the general form of $y=a.b^x$ where a and b are constants, and $b\neq 0$, $b\neq 1$. It is also sometimes written in the form $y=ae^{bx}$ where a and b are constants and e is the natural base of logarithm.

The exponential form of a complex number is

$$r\,e^{i\theta}=r\,(\cos\theta+i\sin\theta).$$

Exponential Series. The infinite power series for exponential function e^x where e is the natural base of logarithm, namely,

$$e^x=1+x+x^2/2\,!+x^3/3\,!+\ldots\ldots x^n/n\,!+\ldots\ldots$$

Hence, $e^{-x}=1-x+x^2/2\,!-x^3/3\,!+\ldots\ldots+(-1)^n x^n/n\,!+\ldots\ldots$

If $x=1$, $e=1+1/1\,!+1/2!+\ldots\ldots+1/n\,!+\ldots\ldots$

and $e^{-1}=\dfrac{1}{e}=1-1/1\,!+1/2\,!-1/3\,!+\ldots\ldots+(-1)^n/n\,!+\ldots$

Expression. A combination of symbols which represent numbers or other mathematical entities. For example, $5x^3$, $3x^2+5 \tan^{-1} x+1-x$. See *Algebraic Expressions, Trigonometric Expressions.*

Extension of Pythagoras's Theorem. The generalization of Pythagoras's theorem to triangles of any shape. According to this extension, in any triangle the square on any side is equal to the sum of the squares on the other sides plus or minus twice the rectangle formed by either of these sides and the projection upon it of the other. Thus, in any triangle *ABC*,

$$a^2=b^2+c^2\pm 2cx$$

where x is the projection of b on c and the plus or minus sign is used according as a is opposite an obtuse angle or an acute angle. This may be written in the cosine form as

$$a^2=b^2+c^2-2bc \cos A.$$

If A is obtuse, cos A will naturally be negative.

Exterior Angle. In case of a polygon an angle at a vertex of a polygon, formed outside the polygon by two adjacent sides, one of which has been extended. The exterior angle and the interior angle are supplementary to each other. In case of a triangle the exterior angle at any vertex is equal to the sum of the angles on the insides of the other two vertices.

Extrapolation. The process of estimating the value of a function or a quantity outside a given range of values. The extrapolation is carried out numerically or graphically. In graphical method a graph is drawn over a known range of values and this curve is extended further. The extension of the curve beyond the known range of values gives estimate of values of the unknown parameters. If the curve is a straight line it is called linear extrapolation. See also *Interpolation.*

F

F.P.S. System. The system of measurements based on the foot, pound, and second as the units of length, mass, and time respectively.

Face. A flat surface of a solid figure. For example, a cube has six faces.

Factor. When a number or an expression is the product of two or more numbers on expressions, each of the latter is called a factor of the former. Thus, 3 and 5 are factors of 15 and $(x+y)$ and $(x-y)$ are factors of (x^2-y^2).

Factorial of a Number. The product of

$$n(n-1)(n-2).....3.2.1$$

is called as factorial n and is denoted by $n!$ It also stands for the number of permutations of n things taken all at a time. If n is large it can be obtained by using *Stirling's formula* :

$$n!=(n/e)^n\sqrt{2\pi n}.$$

Factorization. The process of changing any algebraic or numerical expression from a sum of terms into a product of factors. A number or expression is said to be *prime* if no factors can be obtained of that.

Factor Theorem. $(x-a)$ is a factor of the polynomial $f(x)$ in x if and only if $f(a)=0$. For example, if

$$f(x)=x^2+5x+6=0,$$
$$f(-2)=f(-3)=0$$

so that $(x+2)$ and $(x+3)$ are factors of $f(x)$.

Farenheit Scale. A scale of temperature named after the German instrument maker Daniel Farenheit. The freezing point of water and its boiling point were divided into one hundered and eighty parts. On the Farenheit scale water freezes at 32°F and boils at 212°F. The formula for converting centigrade temperature to Farenheit scale is

$$\frac{C}{5}=\frac{F-32}{9}$$

where C is the temperature on Centrigrade scale and F is on Faranheit scale.

Fallacy. See *Logic.*

Family of Curves. A set of curves with same geometrical pattern. A family of curves is represented by an equation containing one or more parameters. Different values of the parameters give different curves of the family. For example, the equation $x^2+y^2=c^2$, represents a family of concentric circles. Different values of c give different members of the family.

Fermat's Theorem. If n is an integer, p is a prime number, nothing being common in n and p then $n^{p-1}\equiv 1 \pmod{p}$, that is, n^{p-1} divided by p leaves a remainder 1.

Fermat's Last Theorem. The theorem involving three unknown x, y, and z such that $x^n+y^n=z^n$, where n is an integer greater than 2, cannot have any solution in x, y, z. Fermat gave this theorem without proof. Through theorem has been verified and no contradiction has been put forward, proof has not been obtained yet.

Fibonacci Ratios. The ratios of any two consecutive numbers from the *Fibonacci sequence, i.e.*, the ratios 1/1, 1/2, 2/3, 3/5, 5/8, 8/13,..... The limit of the sequence of these ratios is called the *golden ratio* whose value of $(\sqrt{5}-1)/2$. The distribution pattern around the stem of a plant of successive leaves very much fits Fibonacci ratios.

Fibonacci Sequence. The sequence 1, 2, 3, 5, 8, 13,..... in which any term from and after the third, is sum of the last two terms, so that we can write $u_n=u_{n-1}+u_{n-2}$. It's use is found in interpreting Mendelism in the field of genetics.

F-distribution. If pairs of random samples have been drawn from a population and if two samples of a pair are of sizes n_1 and n_2 with variances σ_1^2 and σ_2^2 then the distrbution of the ratio σ_1^2/σ_2^2 is called F-distribution. It is usually used to compare different estimates of the same variance.

Field. A set of elements associated with two binary operations (usually called addition and multiplication) subject to following properties.

(*i*) The set is closed with respect to both the operations.

(*ii*) The set is an abelian group under addition with identi element 0 (zero).

(*iii*) The set is an abelian group under multiplication (ze element excluded).

(*iv*) The two binary operations follow the distributi property, *i.e.*, $a\ (b+c)=ab+ac\ \forall\ a, b, c \in F$, the field. S Subfield, Ordered Field.

Figure. (1) A symbol used to express an integer. (2) shape formed by a combination of points, lines, curves, surfaces. Square, triangles, circles are plane figures, whi spheres, tetrahydrons, cubes are solid figures.

Finite Differences. Consider the function $y=f(x)$. Suppos $y=y_1\ y_2, y_3, \ldots\ldots, y_r, y_{r+1}, \ldots\ldots$. Corresponding to $x=a, a+$ $a+2h, \ldots\ldots, a+(r-1)\,h, a+rh, \ldots\ldots$. The first order diff rences are given by $\Delta y_r=y_{r+1}-y_r$. So that $\Delta y_1=y_2-y$ $\Delta y_2=y_3-y_2$, $\Delta y_3=y_4-y_3, \ldots\ldots$. The second order difference are obtained similarly, so that $\Delta^2 y_r=\Delta y_{r+1}-\Delta y_r$, so tha $\Delta^2 y_1=\Delta y_2-\Delta y_1=y_3-2y_2-y_1$, $\Delta^2 y_2=y_4-2y_3-y_2$, and s on.

If we represent $Ey_r=y_{r+1}$, then $\Delta y_r=Ey_r-y_r=(E-1)$ $\Delta^2 y_r=(E-1)^2 y_r=(E^2-2E-1)y_r = y_{r+2}-2y_{r+1}-y_r$ and i general,

$$\Delta^n y_r=(E-1)^n y_r=[E^n-nE^{n-1}+\frac{n(n-1)}{2}E^{n-2}-\ldots\ldots$$
$$+(-1)^n]y_r$$

$$=y_{n+r}-n\,y_{n+r-1}+\frac{n(n-1)}{2}y_{n+r-2}-\ldots\ldots+(-1)^n y_r.$$

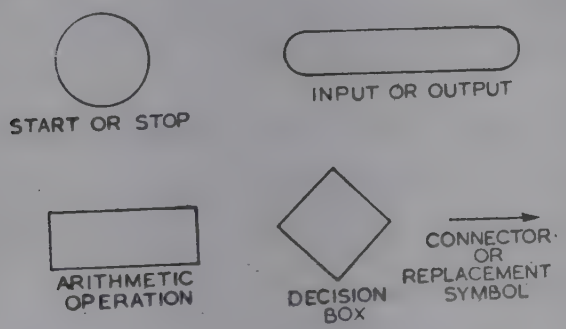
Which is a ready made formula for deriving the nth orde (finite order) differences of any functions.

Finite Group. A group with finite number of elements See *Group*.

Finite Set. A set having a fixed countable number of elements. It can also be taken as a denumerable set with an end number.

Floatation, Law of. An object floating in a fluid displaces fluid equal to its own weight. This follows from the Archimedes' principle for the case of floating objects. A floating object submerged totally or partly in a fluid, is in equilibrium, its only support coming from the fluid.

Flow Chart. A schematic block diagram which indicates the major steps sequentially of a program to be run on a computer. Different operations are represented by different symbols. For example,



Fluid Ounce. A unit of capacity equal to one-twentieth of a pint.

Fly Wheel. A big wheel used in mechanical devices, having large moment of inertia. Energy is used to rotate the wheel at a high speed. Because of large moment of inertia the wheel continues to rotate at a constant speed. The wheel acts like an energy storage device.

Focal Chord. A chord of a conic which passes through a focus. See *Focus, Conic*

Focus. The fixed point associated with a conic. The ratio of the distances of any point on the conic from the focus and the directrix remains the same as the point moves on the conic. See *Conic, Directrix.*

Foot. The unit of length in the F. P. S. system. A foot is one third of a yard. One foot is equivalent to 0.3048 metre.

Foot Pound. (*ft. lb.*) The unit of work in F. P. S. system, that is, the work done in raising a pound mass through one foot against the force of gravitation.

Force. The agent which changes or tends to change the state of rest or the state of motion of a body. The unit force is the force which creates unit acceleration in a body of unit at rest and is a vector quantity as it has magnitude as well as direction. Its' unit is poundal in F.P.S. system and dyne in C.G.S. system and Newton in M.K.S. system.

Force of Gravity. See *Gravitation.*

Forces, Parallelogram (law). See *Parallelogram of Vectors.*

Forces, Triangle (law). See *Triangle of Vectors.*

Format. The arrangement of information on a printed sheet of paper, on a punch card, in a computer device for storage, etc. In other words, it is the lay out of data on the input or output media of a computer system.

Formula. A concise and comprehensive statement expressed symbolically of an equality between different quantities having a precise relation. For example, $V=\frac{4}{3}\ \pi r^3$.

Fortran. Language used in digital computers for scientific and technological purposes.

FORTRAN≡FORMULA+TRANSLATION.

It is a high level problem oriented language.

Four Colour Problem. An unsolved problem of *topology.* Any plane or a spherial surface that is divided into regions may have these regions distinguished if they are coloured in four different ways. This has been shown to be true experimentally but has not been proved rigorously upto now. It appears that in colouring a map it is not necessary to use more than four colours to distinguish regions from each other. Two

regions with a common line boundry between them need different colours, but two regions meeting at a point donot. Seven colours are necessary to distinguish regions on the surface of a torous.

Fourier Series. Any periodic function of period 2π, can be expressed as an infinite trigonometric series of the form

$$f(x)=\tfrac{1}{2}a_0+\sum_{n=1}^{\infty}(a_n\cos nx+b_n\sin nx)$$

where $a_0=\frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,dx$, $a_n=\frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\,dx$,

$$b_n=\frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\,dx.$$

These constants $a_0, a_1, b_1, \ldots, a_r, b_r$, called Fourier Coefficients, represent the amplitudes of the fundamental and successive harmonics which together build up the given periodic function. The above trigonometric series is known as Fourier series.

Fourier Transform. The Fourier transform of a function $f(x)$ is given by

$$F(s)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(x)\,e^{-isx}\,dx$$

Associated with it is the inverse Fourier transform

$$f(x)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} F(s)\,e^{isx}\,ds$$

Fourier cosine and sine transforms of the function $f(x)$ are given by

$$F_c(p)=\sqrt{\frac{2}{\pi}}\int_0^\infty f(x)\cos px\,dx,\quad F_s(p)=\sqrt{\frac{2}{\pi}}\int_0^\infty f(x)\sin px\,dx$$

and the corresponding inverse Fourier sine and cosine transform are given by

$$f(x)=\sqrt{\frac{2}{\pi}}\int_0^\infty F_c(p)\cos pxdp=\sqrt{\frac{2}{\pi}}\int_0^\infty F_s(p)\sin pxdp.$$

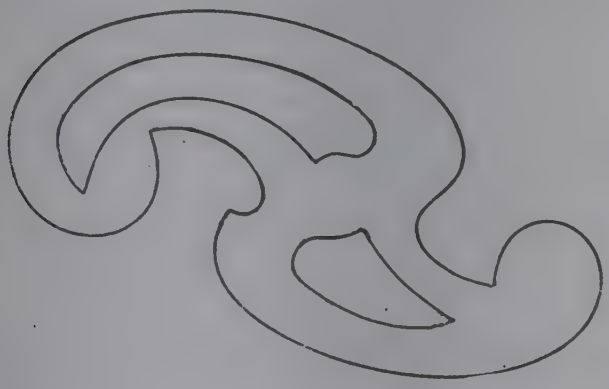
Fraction. A number expressed as a quotient in the form N/D, $(D\neq 0)$, where N is called the *numerator* and D the *denominator*. When N and D are both integers the fraction is called *simple*, *common*, or vulgar fraction. The fraction is known as *proper* if the numerator is less than the demominator, otherwise *improper*. The value of a fraction is unchanged if both the numerator and denominator are multiplied or divided by same constant. The fractions are added by making their denominators as common minimum. For example, $\frac{2}{3}+\frac{4}{7}=\frac{14}{21}+\frac{12}{21}=\frac{26}{21}$. The fractions are multiplied by multiplying the numerators and their denominators separately. For example, $\frac{2}{3}\times\frac{5}{11}=\frac{10}{33}$. Fractions are divided by reciprocating the dividing fraction and then multiply. For example,

$$\frac{2}{3}\div\frac{5}{11}=\frac{2}{3}\times\frac{11}{5}=\frac{22}{15}.$$

If N and D are abgebraic expressions then the fraction is *proper* if the degree of the numerator is strictly less than that of the denominator, otherwise *improper*. The fraction is called complex if both N and D are complex quantities.

Frame of Reference. A set of co-ordinate axes and the origin forming a co-ordinate system with the help of which the position of a point is specified.

French Curve. An instrument used in technical drawing, it made of a piece of metal or plastic sheet with curved edges. The curvature of a french curve varies, from being almost straight line to very right curves so that a part of the edge can be choosen to provide guidance to pen or pencil to move along any desired curvature.



Frequency. (1) The number of times a value or a phenomenon occurs in some unit of time. (2) When a set of observations are grouped into various categories or classes the number of measurements in each class are also known as frequency.

Frequency Curve. The smooth shape of the curve obtained from polygon by joining the mid-points of the top sides of the rectangles forming a *histogram.* Gauss and Laplace gave a bell shape curve called *normal frequency curve.* See *Histogram.*

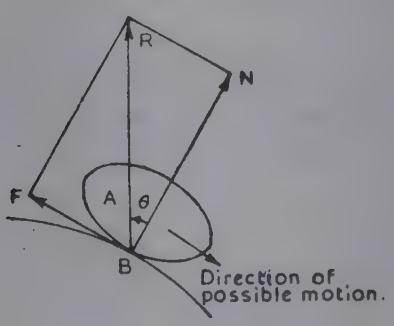
Frequency Distribution. See *Distribution.*

Frequency Function. The function that gives the values of frequency of a variable quantity in an experiment. For a large sample it represents the population. Thus, the observed frequency function will be identical with the probability distribution function of the population. See *Distribution Function.*

Frequency Polygon. The graph obtained by joining mid-points of the top sides of the rectangles in a histog with equal class intervals by line segments. See Histogram.

Frequency Table. A table which shows the frequency the variable occuring in a sample or in an experiment.

Friction. The force that opposes the relative motion the surfaces of two bodies in contact. What happens is each surface exerts a force in a direction opposite to tha relative motion. More precisely, when a body *A*, in con with another body *B*, tends to move towards *B* relatively, external forces in *A* are balanced by a normal reactions *N*, a force *F*, acting on the tangent plane of contact in a direct opposite to that of possible motion. The limiting value of frictional force between two surfaces is called *limiting fric* When the force of friction increases this limiting value of equilibrium gets disturbed and motion starts. The limi value of θ where $\tan\theta = F/N$, is λ which is the maximum valu θ. This λ is known as *angle of friction* and $\tan\lambda = \text{Lt } F/N$ called the *coefficient of friction*, generally denoted by μ.



Frustum. A solid obtained by cutting a solid say co cylinder, prism etc. by two parallel planes.

Fulcrum, The fixed point about which a lever can turn

Function. (1) **Function as correspondence.** A many one or one-to-one correspondence between the elements of

main set X and the elements of *range* set Y. For example, the correspondence between two sets of variables x-the independent variable and y-the dependent variable.

(2) **Function as Mapping**. The set X is the domain of the function f, so that $y=f(x)$, $y\in Y$, Y is the range of the mapping. For every $x\in X$ there is a unique image $f(x)$ in Y.

(3) **Function Relation**. A relation in which to each x in the domain there is only one y. This is a set $\{(x, y)/x\in X, y\in Y, y=f(x)\}$ in which if $y\in Y$ exists for a given $x\in X$, then y is unique.

Functional Analysis. An extension of the study of functions of real or complex variables, in which every set of functions is associated with a unique number. This correspondence is called *functional*. Thus, $\int_a^b f(x)\,dx$, can be interpreted as a functional on the set of all function integrable on (a, b). The value of any functional can be determined only by considering all possible behaviours of the function.

Fundamental Operations. These operations are : addition, subtraction, multiplication, division ; involution, evolution ; rules of indices, logarithms ; differentiation, integration ; inversion, transformation, translation, rotation, reflection. Out of these the first four operations are fundamental operations of arithmetic.

Fundamental Theorem of Algebra. Every polynomial equation of the form

$$a_0z^n+a_1z^{n-1}+a_2z^{n-2}+\ldots\ldots+a_{n-1}z+a_n=0$$

where $a_0, a_1, a_2, \ldots\ldots, a_n$ are all complex numbers, has atleast one complex root.

Fundamental Theorem of Calculus. This theorem provides the method of evaluating a definite integral. If $f(x)$ is

a continuous function of x in $[a, b]$ and if $g(x)$ is indefinite integral of $f(x)$, then

$$\int_a^b f(x)\,dx = \Big[\, g(x) \,\Big]_a^b = g(b) - g(a).$$

Fundamental Units. Units of length, mass, and time denoted by L, M, and T. In C.G.S. system these are Centimetre, Gram and Second ; in M.K S. system these are Metre, Kilogram, and Second ; in F.P.S. system these are Foot, Pound, and Second, almost all the quantities can be expressed in terms of these fundamental units L, M, and T. For example, unit of velocity is LT^{-1}, of acceleration LT^{-2}, of force MLT^{-2}, of stress $ML^{-1}T^{-2}$, etc.

Furlong. Unit of length, one-eighth of a mile or 220 yards which is equivalent to 201.168 metres.

G

g. Symbol stands for (1) acceleration due to gravity, (2) gramme (gram).

Gallon. The British standard unit of capacity for measuring volume of liquids and is defined as the space occupied by ten pounds of water at N.T.P. (normal temperature and pressure). It is equivalent to 4.547 litres.

Gamma Function. Gamma of a number n is defined as

$$\overline{|n} = \int_0^\infty x^{n-1}\, e^{-x}\, dx$$

When n is a positive integer $\lfloor \overline{n} = (n-1)!$ It is easy to see that $\lfloor \overline{1} = \lfloor \overline{2} = 1$, $\lfloor \overline{n+1} = n \lfloor \overline{n}$ and $\left\lfloor \overline{\frac{1}{2}} = \sqrt{\pi}.\right.$

Gauss (*Symbol G*). The unit of magnetic flux density in C.G.S. system.

Gaussian Distribution. See *Normal Distribution.*

General Conic. Family of curves represented by the equation $ax^2+2hxy+by^2+2gx+2fy+c=0$.

Generator. Syn. *Generating line.* A line which generates a surface like a cone or a cylinder.

Geodesic. A line on a surface between two points, which works as the shortest distance between these two points.

Geometric Distribution. The distribution of a number independent Bernoulli trials, before getting a success. For example, the distribution of number of times a coin has to tossed before getting a head. If p and q are the probabilities of success and failure respectively then the probabilities getting of success in k trials is $p(x=k)=q^{k-1}\,p$.

Geometric Means. The terms in a geometric progression lying between the first and the last. In between two numbers a and b we may insert n such means. In this case b becomes the $(n+2)$th term and if r is the common ratio then $b=ar^{n+1}$. thus, the n geometric means in between a and b are

$$a(b/a)^{\frac{1}{n+1}},\ a(b/a)^{\frac{2}{n+1}},\ \ldots,\ a(b/a)^{\frac{n}{n+1}}.$$

If $n=1$ then the geometric mean of a and b is $\sqrt{ab}$. The geometric mean of n quantities $a_1, a_2, \ldots, a_n$ is

$$(a_1 \,.\, a_2 \cdots a_n)^{\frac{1}{n}}.$$

Geometric Progression (G.P.). A sequence in which the ratio of each term to its preceding term remains the same. If the first term is a and common ratio r, then it is written as $a, ar, ar^2, \ldots, ar^{n-1}$ with n terms. The sum of first n terms, S_n, is given by $S_n = \frac{a(1-r^n)}{1-r}$, if $r<1$ and by $S_n = \frac{a(r^n-1)}{r-1}$ If $r \geqslant 1$. Also $S_\infty = \frac{a}{1-r}$.

Geometric Series. A series whose terms form a geometric progression (G.P.). The infinite geometric series is convergent if the common ratio is less than 1 and divergent if the common ratio is greater than or equal to 1.

Geometry. The branch of Mathematics, which deals with lines, curves, solids and surfaces, and points in space. For example, measuring angle between two straight lines or two planes, properties of circles, spheres, relationship between lines and points on a surface. Euclid is considered as father of geometry. Later non-Eluclidean geometry was developed. Descartes in seventeenth Century, gave his algebraic study of geometry by defining axes of coordinates. Thus came analytic geometry which is the combination of algebra and conventional geometry.

Golden Ratio. The ratio $\lambda : 1$ where λ is the positive root of the quadratic $x^2-x-1=0$. Thus, the golden ratio is $\frac{\sqrt{5}+1}{2}$. It can also be written as λ^2/λ or $\frac{\sqrt{5}-1}{3-\sqrt{5}}$. This ratio is the limit of the sequence

$$\frac{1}{1}, \frac{2}{1}, \frac{3}{2}, \frac{5}{3}, \frac{8}{5}, \frac{13}{8}, \ldots.$$

It's approximate value is 1.618. It comes from *golden section.*

Golden Rectangle. A rectange whose adjacent sides are in golden ratio $(1+\sqrt{5})/2$. Every golden rectangle consists of one square and another golden rectangle. The square is formed

with the help of shorter side of the earlier golden rectangle. The process is repeated. The two diagonals drawn intersect at the pole of an equiangular spiral which passes through the centres of the squares.

Golden Section. When a line segment AB is divided internally by a point P in *golden ratio*, the line segment is said to be divided in *golden section*. To achieve it we take $AB=2$ cm and construct a right angle triangle ABC such that $BC=1$ cm. Then obviously $AC=\sqrt{5}$. On AC we take a point K such that $CK=CB=1$ cm then $AK=\sqrt{5}-1$. Now with A as centre we draw an arc PKE such that $AP=\sqrt{5}-1$ and $AE(=\sqrt{5}-1)$ is perpendicular to AB. Then $PB=3-\sqrt{5}$. Here $AP : AB = (\sqrt{5}-1) : 2$. The rectangle $ABDE$ is a *golden rectangle*.

Grad (gradient). Symbol ∇. A vector differential operator operated on a scalar point function. If $\phi(x, y, z)$ is a scalar point function then

$$\text{grad } \phi = \nabla\phi = i\frac{\partial\phi}{\partial x} + j\frac{\partial\phi}{\partial y} + k\frac{\partial\phi}{\partial z}$$

where i, j, k are unit vectors along the coordinate axes. It is a vector with direction in which the rate of change of ϕ is a maximum.

Grade. A unit of plane angle equal to $\frac{1}{100}$th of a right angle.

Gradient (Slope). In rectangular cartesian coordinates, it is the rate of change of y-coordinate of a point on a straight line with respect to x-coordinate. For example, in the equation

$4x-6y+5=0$ the gradient is 2/3, in general $y=mx+c$ has go the gradient as m and c is the length of the intercept on y-axis In case of a curve the gradient changes as the point move along the curve. At a point of the curve the gradient is th gradient of the tangent to the curve at that point. Thus, fo the curve $y=f(x)$ the gradient at (x, y) is $\frac{dy}{dx}$ or $f'(x)$.

Gramme (Gram). The unit of mass in the metric system defined as $\frac{1}{1000}$ of a kilogram.

Graph. The pictorial represention of relationship betwee concerned variables.

Gravitation. The law of attraction discovered by Isaa Newton. According to this law $F=G\,m_1\,m_2/d^2$ where F is th force of attraction between two masses m_1 and m_2 and d is th distance between their centroids. G is the constant of propor tionality known as *gravitational constant*. It was experimentall established that the force of gravitation between two masss o 1 gram distance 1 cm apart is 6.67×10^{8} dynes, so that the valu of $G=6.67\times10^{-8}$.

Gravitational Constant. See *Gravitation.*

Gravitational Field. The region in which a body attracts another body having some mass. The strength of the gravita tional field at any point is given by the ratio of force to mass. This is determined as $G\,m/d^2$, where G is the gravitational constant, m the mass of the object at the centre of the field, and d the distance between the object and the point in question.

Gravity. The force by which the earth (or any other celestial body) attracts a body. See *Acceleration of Free Fall.*

Great Circle. The circle of intersection of the surface of a sphere by any plane passing through its centre. Though the earth is not a perfect sphere even then its equator and all its meridians of longitudes are known as great circles of the earth.

Greatest Lower Bound (g.l.b.). See *Bounds.*

Greenwich Mean Time (G.M.T.). The sun transists through all points on a meridian, hence every point on an E-W line has a different noon time, which gives the local time. The earth may be thought as divided in zones each with a standard time based on the local time of some meredian crossing the zone. Thus, each hour covers a time extending across 15° of longitude. Noon at Greenwich (London) is taken as noon in the British Isles, Portugal, morocco and a part of West Africa. Noon in Central Europe is at 1.00 p.m. G.M.T. (13.00 hours).

Gregory's Series. This is one of the series which yields the value of π and was given by Liebnitz.

$$\pi = 4\left[1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \ldots + (-1)^{n-1}/(2n-1) + \ldots\right]$$

Group. A set of elements S which satisfies the following axioms.

(*i*) It is *closed* with respect to a binary operation *i.e.*, for an operation $*$, $a * b$ belongs to the set S for all a and b belonging to the set S.

(*ii*) The exists an element $e \in S$ called *identity element* such that $a * e = e * a = a$ for any $a \in S$.

(*iii*) There exists an element $a^{-1} \in S$ called *inverse of a* for every $a \in S$ so that $a * a^{-1} = e = a^{-1} * a$.

(*iv*) Set is associative with respect to the operation that is $a * (b * c) = (a * b) * c$ for any $a, b, c \in S$.

The concept of group is of fundamental importance in modern mathematics. Any subset of a group is called a *subgroup* if it itself is a group. A group is said to be *commutative* or *Abelian* if it satisfies the commutative property. A group with finite number of elements is called a *finite* group while a group with infinite number of elements is called *infinite* group. The number of elements in a finite group is called

order of the group. *Symmetric group* is a group of all permutations of n letters. Its order is $n!$. *Alternating group* is a group of all even permutations of n letters. Its order is $n!/2$. *Cyclic group* is a group of elements which are all powers of one element. A cyclic group is always Abelian.

H

Half-angle Formulae. See *Addition Formulae.*

Half-Plane. A part of a plane on one side of a straight line.

Hardware of a Computer. The physical embodiment of a computer system which comprises of its electronic circuits, disk, magnetic or paper tape units, line printer, card reader etc.

Harmonic Mean (H). Harmonic mean of two numbers a and b is given by $H=\frac{2ab}{a+b}$ and that of n numbers $x_1, x_2, \ldots, x_n$ by

$$H=\frac{n}{\sum \frac{1}{x_i}}=\frac{n}{\frac{1}{x_1}+\frac{1}{x_2}+\ldots\frac{1}{x_n}}$$

Harmonic Means. The terms of a harmonic progression lying between the first and the last are called harmonic means of the first and the last terms. To insert n harmonic means between two numbers a and b means to have $(n+2)$ numbers in H.P. The common difference of the reciprocals of the

terms a and b is $\left(\frac{1}{b}-\frac{1}{a}\right)/(n+1)$, and the n harmonics are

$$\frac{1}{a+d}, \quad \frac{1}{a+2d}, \quad \ldots\ldots \quad \frac{1}{a+nd}$$

where d is the above stated common difference.

Harmonic Progressive Motion. See *Simple Harmonic Motion.*

Harmonic Progression. A sequence of terms in which each term is the reciprocal of the corresponding term of a series in *arithmetic progression.*

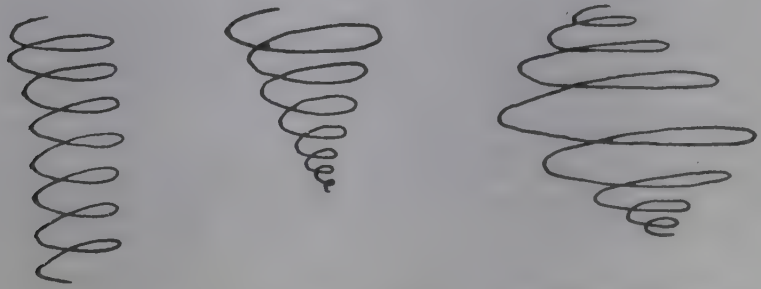
Harmonic Series. The sum of the terms of a harmonic progression. For example :

$$1+\frac{1}{3}+\frac{1}{5}+\frac{1}{7}+\cdots$$

Hecto (Symbol). A prefix denoting 10^2. For example, 1 hecto metres $=10^2$ metres.

Height. Syn. *Altitude.* The vertical distance of a point above the base line or plane. For example, the perpendicular distance from the base of a triangle of the opposite vertex.

Helix. A space curve in the shape of a spiral. It is called cylindrical, cerical, or spherical if it lies on a cylinder, cone, or a sphere respectively. For example, the shape of the thread on a secrew is a helix.



Cylindrical Helix *Cerical Helix* *Spherical Helix*

Hemisphere. Any sphere when intersected by a plane through its centre gives two hemispheres.

Heptagon. A polygon with seven sides.

Heron's (Hero's) Formula. A formula connecting the area of a triangle with its sides. If a, b, c are the length of the sides of a triangle its area $\triangle$ is given by

$$\triangle = \sqrt{s(s-a)(s-b)(s-c)}$$

where $$s = \frac{a+b+c}{2}.$$

s is known as semi-perimeter.

Hexagon. A polygon with six sides. A regular hexagon has all its six sides of equal length and hence all vertical angles equal each being 120°.

Hexahedron. A polyhedron with six faces. Cube is a regular hexahedron.

Highest Common Factor (H.C.F.). The largest number that will divide without any remainder all the numbers whose H.C.F. is being obtained. It is sometimes also known as greatest common measure (G.C.M.)

Hindu-Arabic Numerals. The numbers 1, 2, 3, 4, 5, 6, 7, 8 and 9 together with 0 are very big contribution of Hindu mathematics. Arabs have been at the top in popularizing these numerals. Stevin extended this system to decimal notation in sixteenth century.

Histogram. A graphical representation in statistics in which the width is proportional to the width of the class interval which too is the variable and the height of the rectangular column is proportional to the frequency within that interval. See also *Frequency Polygon.*

Homogeneous Equation. An equation in which a homogeneous function is equated to zero. See *Homogeneous Function.*

Homogeneous Function. An algebraic function in which each term is of the same degree. If each terms is of degree n the function is called homogeneous function of degree n. For example, $f(x, y)=xy+x^2+y^2$ is a homogeneous function in x and y of degree 2. For a homogeneous function $f(x, y, z)$ of degree n, $f(kx, ky, kz)=k^n f(x, y, z)$ for all values of k.

Hooke's Law. In uniaxial case, the law states that the stress is proportional to strain. The pull of spring is proportional to its extension and is given by $T=\lambda \frac{l-l_0}{l_0}$ where l_0 is the original length of the spring. l the length under tension and the constant λ the modulus of elasticity. See *Young's Modulus*.

Horizontal. Parallel to horizon or the level ground. At right angles to the direction of the plumb line (vertical).

Horse-Power (H.P.). A unit of power (rate of doing work) equal to 550 foot pounds per second. It can be interpreted as a mechanical horse, with tractive power of 25 lb and moving at the rate of 15 miles per hour, would exert 1 horse power (H.P.).

Hybrid Computer. It is a combination of analog and digital computer in which advantages of both type have been incorporated. It is mainly used for scientific and technological purposes.

Hydrodynamics. Dynamics of fluids as compared to dynamics of particles and rigid bodies. Thus, in this branch of mathematics we study the motion of liquids and gases.

Hydrostatics. Statics of fluids as compared to statics of particles or rigid bodies. In this branch of mathematics we study the equilibrium condition of liquids and gases.

Hyperbola. A conic with accentricity greater than 1. It has two branches extending upto infinity, two axes of symmetry, two foci, two directrices, two vertices, and two axes *i.e.*, transverse and conjugate and has a centre which is the mid

point of the line of two foci. Transverse axis is the line joining the foci and perpendicular. Conjugate axis is the line through the centre and to the transverse axis. Standard equation of hyperbola is

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$$

where a and b are lengths of semi transverse and semi conjugate axis respectively, the centre being at the origin. Coordinate axes are along the transverse and conjugate axes. Vertices are $(\pm a, 0)$, foci $(\pm ae, 0)$ where $e=\sqrt{1+\frac{b^2}{a^2}}$, the eccentricity. Equations of the directrices are $x=\pm a/e$. It has two asymptotes, $y=\pm\left(\frac{b}{a}\right)x$. If $b=a$ it is a rectangular hyperbola with equation $x^2-y^2=a^2$. If the axes are rotated through an angle $\frac{\pi}{4}$ the equation of the rectangular hyperbola will be $xy=c^2$ in which coordinate axes will be its asympotes. See *Conic*, *Asymptotes*.

Hyperbolic Functions. Functions having an analogy to circular functions, named hyperbolic sine, hyperbolic cosine etc. and defined as

$$\sinh x=\left(\frac{e^x-e^{-x}}{2}\right),\ \cosh x=\left(\frac{e^x+e^{-x}}{2}\right),$$

$$\tanh x=\sinh x/\cosh x,\ \coth x=\frac{1}{\tanh x},$$

$$\operatorname{sech} x=\frac{1}{\cosh x},\ \operatorname{cosech} x=\frac{1}{\sinh x}.$$

These are related to circular functions by the reations

$$\sin(ix)=i\sinh x,\ \cos(ix)=\cosh x \text{ etc.}$$

Also $\cosh^2 x-\sinh^2 x=1$, $\operatorname{sech}^2 x+\tanh^2 x=1$,

$\coth^2 x-\operatorname{cosech}^2 x=1$,

$\sinh 2x=2\sinh x\cosh x$, $\cosh 2x=\cosh^2 x+\sinh^2 x$.

See *Appendix*.

Hyperboloid. A surface obtained by rotating a hyperbola about one of its axes. Rotation about the transverse axis generates hyperboloid of two sheets while rotation about the conjugate axis generates hyperbolid of one sheet.

Hypocycloid. The locus of a point on the circumference of a circle which rolls on the inside of a fixed circle. Its parameteric equation are

$$x=(a-b)\cos\theta+b\cos[(a-b)\,\theta/b],$$
$$y=(a-b)\sin\theta-b\sin[(a-b)\,\theta/b],$$

where a is the radius of the fixed circle, b the radius of the rolling circle and θ the angle between the line of centres and the positive direction of x-axis. There is a cusp of the first kind at every point common to the curve and the fixed circle. The number of cusps is given by $n=a/b$ When $n=4$ the curve is known as astroid whose cartesian equation is

$$x^{2/3}+y^{2/3}=a^{2/3}.$$

See *Epicycloid.*

Hypotenuse. The side apposite to the right angle in a right angled triangle. The ratios of the lengths of the hypotenuse to the lengths of the other sides gives the circular functions sine, cosine, etc. of the angles of the triangles.

Hypothesis. An assumption, a statement or a premise which has yet to tested but is assumed to be true for development of a theory.

I

i. (1) Symbol denoting usually $\sqrt{-1}$. See *Complex* Numbers. (2) In vectors it represents unit vector along x-axis.

Idempotent. See *Algebra of Sets.*

Identity. An equation which is true for every value of the variable or variables involved.

Identity Element. An element of a set which when combined with another element under some binary operation, leaves the latter element as it is. Thus if e is the identity element of a set S under the binary operatlon*. Then $a*e=a=e*a$ for all $a \in S$. For example, in a set of integers 0 is the identity element with respect to the operation of addition and 1 is its identity element with respect to multiplication.

Identity Function. The function $f(x)=x$. Thus when represented graphically gives the straight line $y=x$.

If and only If (iff). See *Algebra of Propositions.*

Image. In mapping, if a set X is mapped into a set Y and $x \in X$ is mapped to $y \in Y$ then y is the image of x.

Imaginary Axis. The axis of y in the *Argand Diagram.*

Imaginary Number. A multiple of i $(=\sqrt{-1})$. Solution of an equation in the form $x^2+2=0$ will need imaginary numbers.

Imaginary Root of Equation. A root of an equation, which is in an *imaginary number.*

Impact. Collision of two bodies where suddenly forces are applied to bodies for a very short time. See *Laws of Impact.*

Imperial Units. Units used in U.K. for a pretty long time. The F.P.S. system was a scientific system based on imperial units. See *F.P.S. Units.*

Implication. (1) Conclusion drawn from a given assumption. (2) Conditional statement of the type, 'If a then b'. See *Algebra of Propositions.*

Implicit Function. If y is a function of x but y cannot be explicity expressed in terms of x then we say that y is an implicit function of x. For example, $x^3+y^3=3axy$.

Implicit Relation. The relation between two variables indirectly their relations with a third variable called parameter. For example, if $x=at^2$, $y=2at$, this is an implicit relation between y and x through the parameter t.

Improper Fraction. A fraction in which the numerator is greater than the denominator. If fraction is composed of algebraic expressions, then the fraction is improper if the degree of the numerator is equal to or greater than that of the denominator.

Impulse (*Impulsive Force*). Product of force and the time of its application, expressed as $\int_0^t Pdt$ which comes out to be $P.t.$ if the force P is constant. Since force is the product of mass and acceleration, or $m.\frac{dv}{dt}$,

then $$\int_0^t P.dt=\int_0^t m\,\frac{dv}{dt}.\,dt=\int_0^t mdv=mv_2-mv_1.$$

where v_1 and v_2 are initial and final velocities. As is known, product of mass and velocity is the momentum, hence the impulse, for instance, in an impact, is measured by the change in momentum produced during impact.

Incentre. The centre of the inscribed circle of a polygon. For example, the incentre of a triangle is the common point of intersection of its angular bisectors.

Inch. An imperial unit of length equal to one-twelth of a foot. Symbol ″.

Incircle (*Inscribed circle*). A circle, if it exists, which touches from insides all the sides of a polygon. Compare *Circum-Circle or Circumscribed Circle.*

Inclination of line in Plane. The angle, called angle of inclination, made by the line with some fixed line in the plane, measured anticlockwise from the fixed line which is generally the x-axis in the plane.

Inclination of Line to Plane. The angle between the line and its orthogonal projection in the plane.

Inclination of two lines. The angle between two lines in a plane

Inclination of two Planes. The smaller angle between the two planes. It is the angle between their normals.

Inclined Plane. A plane which is not horizontal.

Increment. A small positive or negative increase in the value of a variable written as Δ (self variable).

Indefinite Integral. See *Integral of Function.*

Independent Variable. See *Variable.*

Indeterminate Equation. An equation having infinite number of solutions. For example, the equation $ax+by+c=0$ is indeterminate since infinite number of values of x and y will satisfy this equation.

Indeterminate Form. A expression of either of the forms $\frac{0}{0}$, $\frac{\infty}{\infty}$, $0.\infty$, 1^{∞}, $\infty-\infty$ etc. The term indetermi-

ate has become obslete but has relevance only in the sense at these expressions falling in any of the categories cannot be evaluated directly. These are generally obtained by 'Hospital's rule.

Index, Indices. A number indicating a typical characteris- c of an expression. For example, in x^3, 3 is the exponent in the dex, and in $\log_e x$, e is the index.

Induction. A method used in proving that a statement or roposition $P(n)$ is true for all values of n where n is a natural umber. It is done in two steps : (1) $P(1)$ should be shown be true. (2) It should be proved that if $P(k)$ is assumed be true, $P(k+1)$ must be true. If the two steps are establish- l then $P(n)$ is true for all values of n.

Inequality. A statement that a quantity is either greater an or less than but not equal to other. The inequality remains nchanged if a finite positive quantity is either added to or ultiplied on both sides of the inequality. The multiplication r division of two sides by a negative quantity reverses the sign f the inequality.

Inertia. An inherent property of matter, that a body, if at st, will continue to be at rest, and if in motion, will continue move with the same velocity, if no external force is applied to e body.

Inference. A process of reaching a conclusion from a set f premises. There are various rules for drawing inferences. irst rule is of substitution by which the identity

$$(4a)^2-(3b)^2=(4a-3b)(4a+3b) \text{ is obtained}$$

rom the identity $x^2-y^2=(x-y)(x+y)$. Then the rule of de- achment.

Inference in Statistics. Drawing conclusions about a opulation based on the random sample so that the probability f a correct inference can be obtained according to various ypotheses concerning the population under study.

Infinite. Unbounded, not finite.

Infinite Function. A function $f(x)$ such that

$$\lim_{x \to a} f(x) = \infty \text{ or } -\infty$$

is infinite function.

Infinite Series. A series with infinite number of terms.

Infinite Set. A set with infinite number of elements whic can have one-one correspondence with the set of natura numbers.

Infinitesimal. *Variable.* Infinitely small approaching t zero as a limiting value. Concept is used in calculus.

Infinitesimal Calculus. Another name for calculus sinc it is based on the study of infinitesimals. See *Infinitesmal.*

Infinity (∞). An imaginary number denoted by the symbo ∞ which is greater than any number which we can think of. I can also be thought of as $\lim_{x \to 0} \frac{1}{x}$. An infinitely large negativ quantity will be denoted by $-\infty$.

Inflexion. See *Point of Inflexion.*

Initial conditions. The values of the vector quantities like forces, velocities as applicable to a body at the beginning of its movement.

Inner Product. In sets, the intersection of two sets is referred to as inner product of the two sets. In vectors, the scaler product or the dot product is sometimes referred to as inner product of the vectors.

Input. Information fed into the computer in the form of data or instructions. The input device in the computer accepts the input in a coded form and converts it into electrical pulses which are, in turn, transmitted to the central processor. Various input devices are card readers, paper tape readers, terminals which give visual display. See *C.P.U.*

Inscribed Circle. See *Incircle.*

Instanteneous Value. The value of a varying quantity like rate of change, speed, velocity, and acceleration, at a particular instant.

Integer. Any one of the numbers 1, 2, 3..... (positive integers) or -1, -2, -3,..... (negative integers) or zero.

Integer Variable. A variable which can assume integral values only. In programming, any variable name starting with any one of the letters *I, J, K, L, M,* and *N.*

Integral Calculus. Branch of mathematics dealing with integrals of functions.

Integral Curves. A family of curves whose equations are the solutions of a given differential equation. For example, the family of circles $x^2+y^2 = c^2$, which is the solution of the differential equation $x+y\,\frac{dy}{dx}=0$.

Integral Domain. A commutative ring with identity element in it with respect to multiplication and having the property that when $xy=0$ (0 being identity element with respect to addition) then $x=0$ or $y=0$ both $x=y=0$.

Integral Function. Syn. *Entire Function.* A function which when expanded by Maclaurin's series is valid for any assignable value of the variable.

Integral Index. An index (exponent) which is an integer and not a fraction. See *Exponent.*

Integral of Function. Given a function $f(x)$, let there be a function $g(x)$ whose derivative is $f(x)$ then we say that the integral of $f(x)$ is $g(x)$, written as $\int f(x)\,dx=g(x)$. We could also write $\int f(x)\,dx=g(x)+c$ where c is an *arbitrary constant*

of integration and this is the correct way of writing. This known as *indefinite integral* of $f(x)$ since no definite value the integral is known, c being arbitrary. Now, if x lies between two extreme values a and b, then the *definite integral* $f(x)$ over the interval (a, b) is the difference between the value of the indefinite integral of $f(x)$ for $x=a$ and $x=b$, written a

$$\int_a^b f(x)\,dx = \Big[\, g(x)+c \,\Big]_{x=a}^{b} = g(b)-g(a).$$

This represents the *area under the curve* $y=f(x)$ bounded b the curve, the x-axis, and the two ordinates at $x=a$ and $x=b$.

Integrand. The function which is being integrated.

Integration. The process of finding the integral of function.

Integration by Parts. If the integrand $f(x)$ is the produc of two functions $u(x)$ and $v(x)$ then the integral of $(u\ v)$ i given by

$$\int u(x)\,v(x)\,dx = u(x)\int v(x)\,dx - \int\left[\frac{d}{dx}\,u(x)\int v(x)\,dx\right]dx$$

For example, to integrate $(x^2 \sin x)$ we take $u=x^2$, $v=\sin x$, so that the integration by part gives $\int x^2 \sin x\,dx = -x^2 \cos x + \int 2x \cos x\,dx$. Again, integrating $x \cos x$ by parts taking $u=x$, $v=\cos x$, gives

$$\int x^2 \sin x\,dx = -x^2 \cos x + 2x \sin x - 2\int 1.\ \sin x\,dx$$

$$= -x^2 \cos x + 2x \sin x + 2 \cos x.$$

See *Integral of Functions.*

Integration by Substitution. The process of integrating a function by changing the variable of integration in such a

way that the integrand is expressed in a more easily integrable form. For example, to integrate $(1-x^2)^{-3/2}$ a substitution $x=\sin\theta$ will give

$$\int \frac{dx}{(1-x^2)^{3/2}}=\int \frac{\cos\theta\, d\theta}{\cos^3\theta}=\int \sec^2\theta\, d\theta=\tan\theta=\frac{x}{\sqrt{(1-x^2)}}.$$

Intercept. Portion of a line, curve, plane, or a surface cut off by another line, curve, plane, or surface respectively.

Intercept Theorem. If two parallel lines are cut by two transversals, the ratios of the intercepts on one transversal are equal to the ratios of the corresponding intercepts on the other.

Interest. Amount of money paid yearly or half-yearly, or for any period, at a specified rate in terms of percentage, on a borrowed money called capital. The rate is called *rate of interest* and the sum of the principal and the interest after a certain period is called *amount*. This is *simple interest*. See *Compound Interest*.

Interface. Common boundary of two surfaces.

Interior Angle. An angle formed at the vertex of a polygon on the inside of the figure by its two adjacent sides.

Internal Bisector. The bisector of an interior angle of a polygon. It is perpendicular to the bisector of the exterior angle. See *Interior Angle*, *Exterior Angle*.

Internal Division. The division of a line segment internally in a given ratio.

Interpolation. If the values of a function are known at certain points say $x=x_0, x_1, x_2, \ldots, x_n$, the interpolation is the method of estimating the value of the function $y=f(x)$ at a point $x=x_i$ where x_i is different from, $x_0, x_1, x_2, \ldots\ldots x_n$ and is lying in between the extreme values of the given points. If known values of the function are plotted against the given values of x, an estimate of y can be made by drawing a straight line between the two nearest known values. This is known as linear interpolation and the formula for this is

$$y_3=y_1+(x_3-x_1)(y_2-y_1)/(x_2-x_1)$$

where y_3 is the estimated value of y at $x=x_3$ and (x_1, y_1), (x_2, y_2) are the nearest points on the plotted curue. Compare *Extrapolation.*

Intersection. (1) The point at which two or more straight lines, curves, or surfaces meet. (2) For sets, the intersection or inner product of two sets A and B is the set of all those elements which belong to both the sets., denoted by $A \cap B$.

Interval. A set of all the numbers or points lying between two end points. If $x=a$ and $x=b$ are the extreme values then $\{x \mid a<x<b\}$ is called an *open interval* represented by (a, b) whereas $\{x \mid a \leqslant x \leqslant b\}$ is called closed interval represented by $[a, b]$.

Intrinsic Equation of Curve. The equation of a curve in terms of either the radius of curvature ρ and s the arc length or in terms of s the arc length and the angle between the tangent and the positive direction of *x-axis.*

Intrinsic Properties of Curve. Properties of a curve which do not change by the change of coordinate systems.

Invariant. The property of a function, an equation, or a curve which do not change under a particular transformation. For example, in the general equation of a conic

$$ax^2+2hxy+by^2+2gx+2fy+c=0$$

the expressions $(a+b)$ and (h^2-ab) are invariants under the transformations such as translation, or rotation, or both.

Inverse Element. In a set, the inverse of an element a, under the binary operation $*$, is the element a^{-1} such that $a*a^{-1}=a^{-1}*a=e$ where e is the identity element of the set. For example, in the set of integers, $-a$ is the inverse of a since $a+(-a)=(-a)+a=0$, 0 being the identity element with respect to addition.

Inverse Function. If a function f maps a set A into a set B then there may exist a reverse mapping devoted by f^{-1} from the set B into the set A. Thus, if $y=f(x)$ for $x \in A$, $y \in B$ then $x=f^{-1}(y)$. A function that permits such an inverse function

is called *bijective function* and the mapping as *bijective* mapping.

Inverse Hyperbolic Function. Inverse hyperbolic functions are defined analogous to inverse circular functions.

If $y=\sinh x$ then we write $x=\sinh^{-1} y$.

Similarly, the functions $\cosh^{-1}$, $\tanh^{-1}$, $\coth^{-1}$, sech^{-1}, cosech^{-1} may be defined.

Inverse of Matrix. A matrix B of the same order as a square matrix A will be called the inverse of A if $B.\ A=I=A.B$, where I is the unit matrix of the order of A. Here B is written as A^{-1}. Also A^{-1}=adjoint $A/|A|$ where $|A|$ =determinant of A. See *Unit Matrix, Adjoint, Determinant.*

Inverse Number. The inverse of a number n is $1/n$ $(n\neq 0)$.

Inverse Ratio. *Ratio of the inverse.* Inverse ratio of a/b is the ratio of the inverse of a and b, *i.e.*, $(1/a)(1/b)=b/a$.

Inverse Relation. If there is a relation R between two sets A and B so that xRy for $X\in A$, $Y\in B$, then there exists a relation R^{-1} so that $y\ R^{-1}\ x$.

Inverse Square Law. A law according to which the intensity of energy or a force varies as the square of the distance from the source. For example, Force of attraction between any two objects varies as the square of distance between them.

Inverse Trigonometric (circular) Functions.

If $y=\sin x$, then we can say that x is the angle whose sine is y and write $x=\sin^{-1} y$. The function $\sin^{-1}$ is called inverse sine function. Similarly, $\cos^{-1}$, $\tan^{-1}$, $\cot^{-1}$, $\sec^{-1}$, cosec^{-1} are inverse functions corresponding to the trigonometric functions cosine, tangent, cotangent, secant, cosecant respectively.

Inverse Variation. A variation of the form $y=k/x$ where k is a constant of proportionality. We say, y varies inversely with x.

Inversion. If AB is a chord of a circle of radius r then a point C on AB extended, such that $AC.BC=r^2$, then C is called the *inverse* point of B about A and B is the inverse point of C about A.

Involute. Involute of a curve is another curve which is obtained by unwinding a taut-string around the first curve.

Irrational Numbers. A number which cannot be expressed as a ratio of two integers. The transcedental numbers like e and all surds like $\sqrt{5}$ are irrational numbers.

Iso-. Derived from the Greek word *isos* which means *equal* when lines are drawn on the map through places experiencing equal intensity of a natural phenomenon, these are known as *isobars* (atmospheric pressure), isobaths (depth of sea—bed), isotherms (temperature) isoseismals (earthquake shock) etc.

Isolated Point. Syn. *Acnode.* A point which satisfies the equation of a curve but is not on the main arc of the curve. For example, origin is a isolated point of the curve represented by the equation $y^2(x^2-1)=x^2$.

Isometry. A transformation in which the distance between two points remains unaltered.

Isomorphism. A one-one correspondence between the sets, in which each element of any one set has an image in the other set under the above mapping. For example, the one-one correspondence in the form of multiplication by a constant integer is an isomorphism from one set of integers to another set of integers.

Isosceles. Having two sides equal. A triangle with two equal sides is called *isosceles triangle*, a trapezium with its non-parallel sides as equal is called isosceles trapezium.

Iteration. A method of solving a problem by successive approximations in such a way that each approximation comes out with a more accurate estimate by using the preceeding approximation.

J

J. It usually denotes a unit vector in the positive direction of y axis

Job. A unit of work to be run on a computer. Generally one job includes a few programmes. The necessary instructions to run a job on a particular computer installation form a part of input and are fed in the job control language (JCL) of that installation. The JCL is interpretted by the computer and is used to identify the job and specify its requirements to the computer installation.

Join. A line segment joining two given points.

Jordan Curve. A continuous, closed curve not intersecting itself. It may or may not have a finite length.

Jordan Curve Theorem. A Jordan curve divides the plane into two regions having the curve as common boundary.

Joule. A unit of work in MKS system and is equal to the work done by a force of one newton when its point of application moves by one metre in the direction of the force. Its symbol is J.

K

Kelvin Scale. Syn. *Absolute Scale of Temperature* : named after the British mathematician Kelvin. This scale does not depend upon the properties of any substance. The *Absolute Zero* 0°k is equal to $-273.10°$ celsius or centigrade which is assumed to be temperature of interstellar space. One Kelvin is equal to one degree of Celsius or Centigrade.

Kepler's Laws. Laws of planetary motion given by Kepler. (1) Every planet moves in elliptic orbit, Sun being at one

of its foci. (2) The radius vector, *i.e.*, the line joining a planet with the Sun, sweeps out equal areas in equal interval of times (3) The square of the period of revolution of each planet is proportional to the cube of the semi major axis of the ellipse, the mean distance of the planet from the Sun.

Kernel. If is often required to obtain the solution of a linear differential equation in the form

$$y(z)=\int k\,(z,\,t)\,G(t)\,dt$$

Here $k\,(z,\,t)$ is called the *Kernel*. One useful possibility occurs if $k(z,\,t)=e^{zt}$.

Keypunch. The machine used to punch the cards. By pressing the keys on the punching machine, small rectangular holes are punched on the card. Thus the information is fed on the punch cards through keypunch in coded form, which, in turn, is passed to the computer through card reader.

Kilo. In metric system it is a prefix denoting 1000 with symbol K. For example, 1 Kilogram (Kg) = 1000 grams, 1 Kilometer (Km) = 1000 metres, 1 Kilowatt (KW) = 1000 Watt.

Kilowatt-hour. An electrical energy production or consumption of 1000 watt per hour, equivalent to 3.6×10^6 Joules.

Kinematics. The study of various modes of motion of objects.

Kinetic Energy. The energy of a moving object because of its motion. If the velocity of a body of mass m is V at some instant, its kenetric energy is $\frac{1}{2}mV^2$. In some time if the velocity changed from V_1 to V_2 the change in kinetic energy is $\frac{1}{2}m\,(V_2^2-V_1^2)$.

Kinetics. The study of motion of objects in dynamics in relation to the forces responsible for the motion.

Kite. A quadrilateral with two pairs of equal sides and hence only one pair of equal angles. Its diagonals cross each other at right angle, the shorter diagonal being bisected by the longer one. The area of the kite is equal to the product of its diagonals.

Klein Bottle. A curved topological surface in the form of a bottle having the typical characteristic, that it has only one surface with no edges and no inside and outside. See *Topology*.

Knot. A speed of one nautical mile per hour. See *Nautical.*

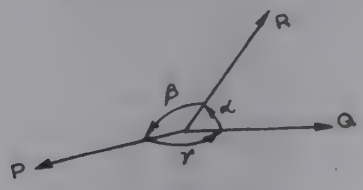
Konigsberg Bridge Problem. An age old classical problem of Topology. In the Prussian City of Konigsberg a river which was divided into branches and had seven bridges on it, was to be crossed in single stretch of journey. It was solved by Euler by replacing the arrangement by a diagram having lines and vertices, called topological graph with points for mutually exclusive land areas and line segments for bridges. He showed that this graph can be traversed in a single journey if and only if there are less than three vertices at which an odd number of line segments meet. In this case there are four.

Kurtosis. A measure of the degree of peakedness of a distribution given by μ_4/σ^4.

L

Lamina. A solid with two parallel faces separated by a very small distance. This concept is very much used while finding the centroids of areas.

Lami's Theorem. If three forces act upon a body to keep it in equilibrium the forces must be coplanar and each force will be proportional to the sine of the angle between the lines of action of the other two.



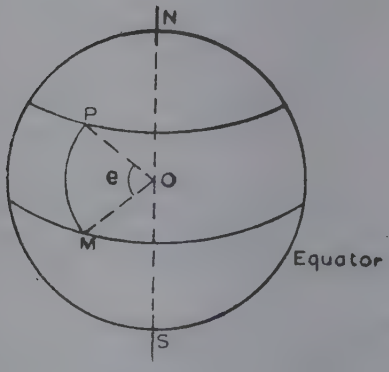
Thus $\frac{\vec{P}}{\sin \alpha} = \frac{\vec{Q}}{\sin \beta} = \frac{\vec{R}}{\sin \gamma}$.

Language. Short form of *programming language*. See *Program.*

Laplace Equation. See *Partial Differential Equation.*

Lateral. It refers to the side of a solid of a geometrical figure other than the base. For example, a lateral edge of a pyramid is one of the edges drawn from its top vertex ; and a lateral face of a pyramid is any face other than the base.

Latitude. The shortest distance of a point P on the earth's surface from the equator measured as PM along the surface of the earth, as an angle θ subtended at the centre of the earth.



The north pole N and south pole S have latitude 90° and any point on the equator will obviously have 0° latitude.

Latus Rectum. Double ordinate through the focus of a conic section. Half of it is known as semi-latus rectum. For example, length of the latus rectum of the parabola $y^2=4ax$ is $4a$ and that of ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}-1$ is $\frac{2b^2}{a}$.

Laws of Consveration. See *Conservation of Energy, Conservation of Mass, Conservation of Momentum.*

Laws of Gravitation. See *Newton's law of Gravitation.*

Laws of Impact in Elasticity. If two balls collide while moving in one direction with velocities u_1 and u_2 ($u_1 > u_2$ say) and after the impact their velocities are v_1 and v_2 ($v_1 > v_2$) then,

$$v_1 - v_2 = e\,(u_1 - u_2),$$

where e is called the *coefficient of restitution* which depends upon the elasticity of the materials the balls are made of.

Laws of Impact in Rebound. If a ball falls from a height h on a horizontal plane and rebounds to a height h_1 then $h_1 = e^2 h$ where e is the *coefficient of restitution.*

Laws of Motion. See *Newton's Laws of Motion.*

Leading Diagonal. See *Square Matrix.*

Least Common Denominator. See *Common Denominator.*

Least Common Multiple (L.C.M.). The smallest common multiple of numbers or of polynomials whichever is the case.

Least Square Method See *Method of Least Squares.*

Least Upper Bound. (l.u.b.) See *Bound of a Set, Bound of a Function.*

Legendre Polynomials. A series of functions $P_n(x)$ in the form of polynomials, which are solutions to the Legendre's differential equatian. See *Legendre's Equation.*

Legendre's Equation. The differential equation of second order of the form

$$(1-x^2)\,\frac{d^2y}{dx^2} - 2x\,\frac{dy}{dx} + (x^2 - n^2)\,y = 0.$$

Leibnitz's Theorem. A rule for finding the nth derivative of the product of two functions. It is expressed as a series of $(n+1)$ terms involving successive differntials of separate functions and binomal co-efficients, as follows :

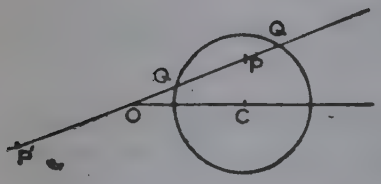
$$D^n(uv)=(D^nu)v+n(D^{n-1}u)\,(Dv)+\frac{n(n-1)}{1.2}(D^{n-2})\,(D^2v)+$$

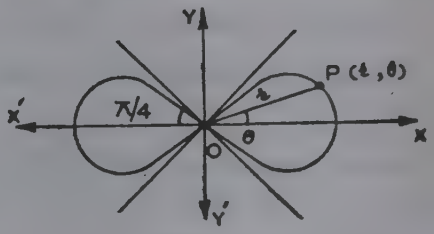
$$+\frac{n(n-1)\,(n-2)}{1.2.3}\,(D^{n-3}u)\;D^3v+\ldots\ldots+u(D^nv)$$

It is quite similar to binomial series for positive integral index.

Lemma. A theorem sort of thing which has already been proved and is used as a basic assumption in the form of axiom or premise, in another proof.

Leminiscate of Bernoulli. Let O be a point at a distance $a\sqrt{2}$ from the centre C of a fixed circle of radius a and take a variable secant through O which meets the circle in Q and Q'. Now the locus of two points P and P' on OQ such that $OP = OP' = QQ'$ are two branches of the curve called





Leminiscate given by Bernoulli. Its cartesian equation is $(x^2+y^2)^2=a^2(x^2-y^2)$ and the more convinient is the polar equation $r^2=a^2\cos 2\theta$.

Length. The distance along a line or a plane. In a rectangle the greater side is known as length while the smaller one as breadth. Its unit is metre or centimetre in C G.S. system while it is foot or yard in F.P.S. system.

Lever. A class of machines or a rigid objects able to turn around some fixed point called fulcrum. These are of, three types.

First Type. In these the fulcrum is between the load and the effort. For example, a crowbar.

Second Type. In these the load is between the fulcrum and the effort. For example, a wheelbarrow.

Third Type. In these the effort is between the fulcrum and the load. For example, a tunning fork.

For a lever of any type to be in equilibrium, the algebraic sum of the three moments about any point must also be zero.

Library Program. The built-in programs for ready use in a computer. These are used in the form of function subprograms. A good computer installation must have a rich library. For example, library programs for finding the inverse of a matrix, for numerical integration by Simpson's rule, etc.

Light Year. A unit of distance for use in astronomy. It is defined as the distance travelled by light in space in one full year. It is approximately equal to 9.467×10^{12} Km.

Limacon. In plane it is the locus of a point on a line at a fixed distance from the point of intersection of the line and a fixed circle, as the line rotates about the fixed point on the circle. The equation of limacon is $r = b + 2a \cos \theta$ where a is the radius of the fixed circle and b the fixed distance. If $b < 2a$ there are two loops. If $b > 2a$ there is one loop which approaches to the shape of a circle as a increases. When $b = 2a$ the curve is cardioid with cusp at the pole. The curve was given by Pascal.

Limit. Generally, the value to which a variable or a function approaches.

Limit of Class Interval. See *Class Limits.*

Limit of Function. A quantity l is said to be the limit of a function $f(x)$, written as $\lim_{x \to a} f(x) = l$

if, for a given small positive number ϵ there is a number δ such that for every $|x - a| < \delta$, $|f(x) - l| < \epsilon$.

By *Fundamental Theorem on limits,* for x approaching to a, $\lim (u \pm v) = \lim u \pm \lim v$, $\lim (u.v) = \lim u . \lim v$,

$$\lim \left(\frac{u}{v}\right) = \frac{\lim u}{\lim v} \text{ for } \lim v \neq 0.$$

Limit of Sequence. The limit of a sequence $\{u_n\}$ is th limit of u_n as n tends to infinity. More precisely, l is the limi of the sequence $\{u_n\}$ written as $\lim_{n\to\infty} u_n = l$, if, given a smal positive number ϵ there exists a number N such that $| l - u_n | <$ for all n greater than N.

Limit point. Syn. *Accumulation Point.*

Limits of Integration. See *Definite Integration.*

Line. Short form of *straight line.*

Line of best Fit. (1) A trend line (2) A line obtained by applying the method of least squares.

Line Integral. (Contour Integral, Curvilinear Integral). The integral of a function carried out over a particular path C which is either a line segment or a portion of a curve. When the path C is a closed curve, the line integral is known as closed integral.

Line Printer. An output device of a computer, which prints the desired results on a paper line by line. It does the o b very fast.

Line Segment. A finite portion of a straight line.

Linear. (1) Having the properties of a straight line, (2) One dimensional, (3) Of first order (4) Of first degree.

Linear Algebra. Branch of mathematics in which algebraic properties of vector spaces are studied. If V is a set of vectors $V = \{v_1, v_2, v_3, \ldots\}$ and F a set of scalars $F = \{k_1, k_2, k_3, \ldots\ldots\}$, V can be subjected to the operations of vector additions $(v_1 + v_2)$ etc. and operations of scalar multiplication $(k_1 v_1, k_2 v_2)$ etc. or both $(k_1 v_1 + k_2 v_2,)$ etc. then results of such operations are vectors of linear form.

Linear Combination. If $v_1, v_2, v_3, \ldots v_n$ are certain quantities and $k_1, k_2, k_3, \ldots\ldots, k_n$ are constants then the quantities

$k_1v_1+k_2v_2+\ldots\ldots+k_nv_n$ form a linear combination in which all the $k's$ are not zero.

Linear Dependence. A set of quantities q_1, $q_2,\ldots\ldots,q_n$ which may be polynomials, matrices, vectors etc. are called linearly dependent if there are constants $k_1, k_2,\ldots\ldots, k_n$, not all zero, such that $k_1q_1+k_2q_2+\ldots\ldots+k_nq_n=0$.

Linear Differential Equation. A differential equation of first degree in y and its derivatives, where the coefficients of y and its derivatives are functions of x only.

Linear Displacement. Syn. *Translation.*

Linear Equation. First degree equation. For example, $ax+b=0$, $a\neq0$.

Linear Expansion. Expansion one directional.

Linear Extrapolation. See *Extrapolation.*

Linear Interpolation. See *Interpolation.*

Linear Programming. (*Linear Mathematical Programming*). This branch of mathematics deals with the problem of finding maximum or minimum values of a linear function under limiting conditions. For example, the problem of finding the maximum value of the linear function $x+y$ subject to the conditions $x\geqslant1$, $y\geqslant\frac{1}{2}$, $5x+6y\leqslant30$, $3x-y\geqslant3$, is the problem finding the point within the region satisfying the above conditions and at a maximum distance from the line $x+y=0$.

Linear Transformation. A transformation obtained by the use of linear equations. For example, transformation of axes.

Literal Coefficients. Coefficients composed of literal constants like a, b, c. For example, the coefficient in the expression $ax^4+bx^3+cx^2+dx+e$, are literal coefficients.

Literal Constants. A letter such as a, b, c, etc. used to denote a constant, in contrast to numerical constants like 1, 2, 5, 7, etc.

Literal Equation. An equation in which the coefficien are literal constants. For example, the equation $ax^2+bx+c=$

Literal Expression. An expression in which the coef cients are literal constants.

Lissajous Figures. Resulting pattern of two prependicul simple harmonic motions.

Litre. A unit of volume in metric system, equal to 100 cubic centimetres. It can be taken as the volume of one kg. pure water at 40°C and normal pressure.

ln. Symbolic of logarithm of a quantity to the base e s that $ln\ x=\log_e x$.

Local Maxima. A value of the variable x for which th value of the function $f(x)$ is greater than the values of $f(x)$ in th neighbourhood of that value of x. There can, naturally, b more than one local maxima of any function.

Local Minima. A value of the variable x for which th value of the function $f(x)$ is less than the values of $f(x)$ in th neighbourhood of that value of x. There can, naturally, b more than one local minima of any function.

Locus. The path traced out by a point constrained t move in accordance with given conditions. The locus may b a point, line, curve, plane, a surface, or a solid. For exampl the locus of a point which moves in such a way that its distanc from a fixed point is always equal to its distance from a fixe straight line, is a parabola.

Logarithm. A number which may be expressed as th exponent (power) of another number called its base. Thus, th solution of $a^x=N$ where a and N are known, is called logarithm the base of N to a, written as $x=\log_a N$. The inverse relation "the antilogrithm of x to the base a is N". The rules of opera tion of logarithm are derived from theory of indices and ar given by $\log_a (x\ y)=\log_a x+\log_a y$, $\log_a (x/y)=\log_a x-\log_a$ The rule for the change of base is

$$\log_a N=\log_b N/\log_b a=\log_b N\ .\ \log_a b.$$

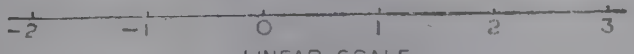
Common bases, in use, are 10 and e ($\equiv$2.71828) is called th natural base of logarithm, the corresponding logrithm is know as *Natural logarithm* or *Napierian logarithm* given by Napier, an

s written as ln x or $\log_e x$. The integral part of logarithm is called *Characterstic* and the decimal part as *mantissa.*

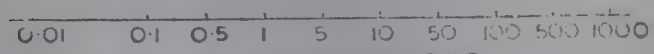
Logarithmic Curve. The curve with equation $y=\log_e x$. The image of this curve about the line $y=x$ is the exponential curve $y=e^x$.

Logarithmic Function. If two variables x and y are connected by the relation $y=\log_a x$ where the base a is positive then y is said to be logarithmic function of x.

Logarithmic Scale. A line in which the distance x from some fixed point on it, is proportional to the logarithm of a number. For example, one unit of length along the line represents 10, two units 100, three unit 1000 and so on ; negative one unit represents 1/10 negative two units 1/100, and so on.



LINEAR SCALE



LOGARITHMIC SCALE

In this case the distance x along the logarithmic scale is given by $x=\log_{10} a$. The slide rule is formed on the basis of logarithmic scale since two numbers may be multiplied (divided) by adding (subtracting) on a logarithmic scale.

Logarithmic Series. The power series used to calculate natural logarithm of a number, namely,

$$\log(1+x)=x-x^2/2+x^3/3-x^4/4+\cdots, \qquad |x|<1,$$

and $$\log(1-x)=-[x+x^2/2+x^3/3+x^4/4+\cdots].$$

Logarithmic Spiral. Syn. *Equiangular Spiral.*

Logarithmic Table. A table showing mantissa *i.e.*, the fractional parts of logarithms of numbers.

Logic. The study of principles and methods on the basis of sound arguments and reasoning. We study whether the process by which a conclusion drawn from a set of initial assumptions is correct.

Log-Linear Graph (*Semi-logarithmic graph*). A graph
which one axis has logarithmic scale and the other has a line
scale. The graph of a curve having the equation of the for
$y=ae^{bx}$ will be a straight line where a and b are constant
See *Logarithmic Scale.*

Log-Log Graph. A Graph on which both the axes ha
logarithmic scales. See *Logarithmic Scale.*

Longitude. The east-west location of a point on the earth
surface, measured as an angle in degrees from Greenwhi
Meredian (a standard G.M.T. meridian) See *Meridian, Gree
which Mean Time* (G.M.T.).

Loop. A set of instructions in a computer program, whi
is executed repeatedly until some specific requirement
achieved.

Lower Bound. See *Bound of a Set.*

Lowest Common Denominator. Syn. *Least Comm
Denominator.*

Lowest Common Mulliple. Syn. *Least Common Multipl*

M

M.K.S. System. The system of measurement in Metr
Kilogram, and Second as units of length, mass and tim
respectively. See *Metric System.*

Machine Language (Machine Code). In computer pr
gramming all the data and instructions are supplied to th
central processor in a binary code (machine code) with only tw
digits 0 and 1. Thus, the machine language is all in terms

binary digits only and though it is possible for the programmer to write his program in machine language, it is a quite tedious and time consuming job. Therefore the programs are normally written in high level languages and then automatically translated to machine language with the help of compilers.

Maclaurin's Series. (Theorem or expansion) If a function $f(x)$ is continiously differentiable any number of times in the neighbourhood of the origin then

$$f(x)=f(0)+xf'(0)+\frac{x^2}{2!}f''(0)+\frac{x^3}{3!}f'''(0)+\ldots\ldots$$

See *Taylor's Series.*

Magic Square. A square array of order $n\times n$ developed from the integers 1, 2, 3,....., n where n is the order of the magic square. The magic square is formed in such a way that the sum of the elements in any row, column or diagonal is the same. The three basic squares are given below.

2	9	4
7	5	3
6	1	8

12	7	9	6
13	2	16	3
8	11	5	10
1	14	4	15

9	2	25	18	11
3	21	19	12	10
22	20	13	6	4
16	14	7	5	23
15	8	1	24	27

The elements of such arrays can be rearranged also. There in only 1 magic square of orders 3,880 of orders 4 and an unknown number of magic squares of order 5.

Magnetic Disk. It is widely used device for computer systems for storing information. It is a flat circular metallic plate coated on both sides by a magnetised material. On the coated surface there are concentric tracks having spots magnetised in one or two directions so that the information is binary. One disk can store several million characters. Information can be added, altered or deleted as required by magnetic means.

Magnetic Tape. A long plastic strip coated with magnetic substance on which information is stored. Magnetic Tapes as used in cassets is known by everybody but this is also very widely used in computers to store information which is to be fed into the computer or taken out of the computer. On the magnetic tapes there are small closely packed spots in which information gets stored. A magnetic tape is capable of storing a very huge amount of information which can be deleted, altered, or added also. A tape is generally half-inch wide and up to 700 metres long.

Magnitude. The *absolute value* of a real or complex number or a vector.

Major, Minor arc of Circle. An arc greater or less than half the circumference of a circle.

Major, Minor Axis. See *Ellipsoid.*

Major, Minor Sector. A sector of a circle (sphere) greater than or less than a semicircle (hemisphere).

Major, Minor Segment. A segment of a circle (sphere) greater than or less than that of a semicircle (hemisphere).

Manifold. A class with subclasses. For example, a plane is two dimensional manifold of points as each point is specified by two co-ordinates, the abscissa and the ordinate. Similarly, Euclidean space is a three dimensional manifold of points.

Mantissa. See *Logarithms*

Mapping. Syn. *Function, Transformation,* and *Correspondence.* See *Function.*

Mass. The quantity of matter contained in a body. The unit of mass is gramme in C.G.S. system, kilogram in M.K.S. system, and pound in F.P.S. system.

Mass, Centre of. See *Centre of Mass*

Mass-Energy Equation. The equation $E=mc^2$, where m is the mass of a body, c the speed of light, and E the total energy of the body.

Mathemetical Induction. See *Induction.*

Mathemetical Programming. The process of finding an optimum value *i.e.* a maximum or a minimum value of a function when the variables involved in the function, are subjected to certain conditions in the form of equations and inequalities. The programming is known as linear or non-linear according as the function is linear or non-linear respectively.

Matrix. An array of numbers, called elements, arranged in the form of rows and columns. The matrices do not have any numerical value unlike determinants. Vectors may be represented by row or column matrices. A set of simultaneous algebraic equations can be solved more conveniently with the help of matrices. Cayley developed the algebra of matrices.

A matrix with m rows and n columns $(m \neq n)$ is called a *rectangular matrix of order* $m \times n$ and a matrix with number of rows equal to number of columns $(m=n)$ is a called *square matrix* of order $n \times n$. *Zero matrix* is one in which all the elements are zero. A matrix may be denoted by a capital letter like $A=(a_{ij})$, where a_{ij} is its element in ith row and jth column. A square matrix A will be called *a diagonal* matrix if $a_{ij}=0$ for $i \neq j$ and a *lower (upper) triangular* matrix if $a_{ij}=0$ for all $i>j$ $(i<j)$. The square matrix A is a *unit* (identity) matrix if $a_{ij}=0$ for $i \neq j$ and $a_{ii}=1$. Two matrices of the same order can be added by adding the corresponding elements. Multiplication of a matrix by a scalar is given by $C=kA$ in which $C_{ij}=ka_{ij}$ for all i, j. Matrices A and B can be multiplied to form the product AB if the number of columns in A [illegible] same as number of rows in B. If $A=(a_{ij})_{l \times m}$, $B=(b_{ij})_{m}$ [illegible] $=(c_{ij})_{l \times p}=AB$

where $c_{ij}=\sum_{k=1}^{m} a_{ik} b_{kj}$. for $i=1,2,\ldots\ldots, l$ and $j=1, 2,\ldots\ldots, p$.

The matrix multiplication is not commutative.

Maxima, Minima. The necessary condition for a function $y=f(x)$ to have a maxima or minima at $x=a$ is $f'(a)=0$. Further, $x=a$ has a maxima or minima if $f''(a)$ is negative or positive. More correctly, $f(x)$ in such situations is said to have a local maxima or minima at $x=a$.

Mean (Average). The arithmetic mean or average of $x_1, x_2, \ldots\ldots, x_n$ is equal to $(x_1+x_2\ldots+x_n)/n$. If $x=x_1$ has a frequeney f_1, frequency f_2, for $x=x_2$ and so on then the arithmetic mean or average has a rather *weighted mean* equal to

$$(f_1x_1+f_2x_2+\ldots+f_nx_n)/(f_1+f_2+\ldots+f_n).$$

Of the quantities $x_1,x_2,\ldots\ldots, x_n$ the geometric mean G is given by $G=(x_1,x_2.x_3\ldots\ldots x_n)^{1/n}$ and the harmonic mean H is given by $H=n\Big/\left[\frac{1}{x}+\frac{1}{x_2}+\ldots\ldots+\frac{1}{x_n}\right]$.

Mean Deviation. It is a measure of dispersion of elements of a set from its mean or average. If $\bar{x}$ is the mean of $x_1,x_2,\ldots\ldots, x_n$ then the mean deviation of x from $\bar{x}$ is equal to

$$\sum_{i=1}^{n}(x_i-\bar{x})/n.$$

Mean Value Theorem in Differential Calculus. If a function $f(x)$ is continuous and is continuously differentiable, that is $f(x)$ is differentiable and $f'(x)$ is continuous over the closed interval $[a,b]$ then there exists a value c of x in (a, b) such that $f(b)-f(a)=(b-a)f'(c)$. This is called *Rolle's Theorem.*

Mean Value Theorem in Integral Calculus. If a function $f(x)$ is continuous and single valued over the closed interval $[a,b]$, then there exists a value c of x in $[a, b]$ such that

$$\int_a^b f(x)\,dx=(b-a)\,f(c).$$

Mechanics. A branch of mathematics in which the effect of forces on an object, is studied. When the body is in equilibrium under the action of forces the subject is called *Statics* and when the body is in motion the subject is called *Dynamics*, When *a* body is at rest or is in motion inside a fluid the mechanics is known as *Fluid Mechanics.*

Median. (1) In Statistics, it is the middle element of a set of measurements arranged in the order of size. When there are even number of elements the median is taken to be the mean of the middle two elements.

(2) In Geometry it is a straight line joining the vertex of a triangle to the midpoint of the opposite side. Medians of *a* triangle meet at a point called *centroid.*

Mega. Symbol 'M'. This denotes a prefix equal to 10^6.

Member. See *Element.*

Memory. See *Storage.*

Mensuration. A branch of Mathematics dealing with the measurement of geometrical figures and finding their length, area, and volume.

Meridian. A *Great circle* passing through the north and south poles. A local meridian of a point on the earth's surface is the great circle passing through the north and south poles and that point.

Method of Least Squares. A method for determining the best representative value for a set of data under the presumption that this occurs when the sum of the squares of the deviations is a minimum.

Metre. The unit of length in metric system, the M.K.S. system, equal to 100 centimetres.

Metre, Kilogram, Second System (M.K.S.) System. System of units of measurements in Kilogram for mass, Metre for length and second for time.

Metric Space. Any set of points in which a metric $d(a, b)$ in between any two elements a and b (*e.g.*, distance between the two points a and b) is defined so as to satisfy the following properties.

(*i*) $d(a, b) \geqslant 0$, (*ii*) $d(a, b) = 0$ if and only if $a = b$.

(*iii*) $d(a, b) = d(a, c) + d(c, b)$.

For example, set of all continuous function in (a, b).

Metric System. A system of units of measurements based on metre, kilometre, centimetre, milimeter for length ; kilogram,

gram, miligram for mass ; litre, mililitre for volume, etc. Or units using multiples and submultiples of 10. S.I. units, C.G.S. units, M.K.S. units all come under metric system.

Metric Ton. See *Tonne.*

Micro. Symbol μ. A prefix denoting 10^{-6}. For example, 1 micrometer $= 10^{-6}$ metre. See *Metric System.*

Micro Processor. See *Central Processing Unit.*

Mile. British unit of length equal to 1760 yards $\simeq$ 1.6093 km.

Milli. A prefix denoting 10^{-3}. For example, 1 milimetre $= 10^{-3}$ metre $= \frac{1}{10}$ centimetre.

Millibar (m.b.). The unit of atmospheric pressure used on weather charts.

Minor Arc of a Circle. See *Major, Minor Arc of circle.*

Minor Axis. See *Major, Minor Axis.*

Minor in Determinant. The minor of an element in a determinant is that determinant of order one less than that of the original determinant, obtained by deleting the row and column in which that element occurs. The minor of an element a_{ij} multiplied by $(-1)^{i+j}$, is called *cofactor* of the element a_{ij}.

Minor Sector. See *Major, Minor Sector.*

Minor Segment. See *Major, Minor Segment.*

Minute. As unit of time it is 1/60th of an hour, or equal to 60 seconds. As a unit of plane angle it is equal to $\frac{1}{60}$ th of a degree.

Minus. A Latin word for less. It has the symbol $-$ and when put before any number of an expression it signifies a negative quantity as one 'subtracted' from zero. For example, $-37 = 0 - 37$.

Mobius Band (Strip). A continuous flat loop with one twist in it. If a flat rectangular strip $ACBCD$ is joined so that D fits B and C fits A by giving the strip a twist in the middle and a Mobius strip is obtained with one continuous surface. Here the pecularity is that any point on the original strip can be joined to any point on the other side without crossing an edge as it has one edge and one surface only. This is very important in *topology*.

Modal Class. The class with highest frequency in a grouped data. See *Mode*.

Mode. The value of a variable, which has the highest frequency in a series of observations.

Modulo (Modular) Arithmetic. The modulo n or mod n arithmetic uses only the numbers $0, 1, 2, \ldots, n-1$. When the fundamental operations are carried out and the number exceeds $(n-1)$ it is divided by n and the remainder is used instead of the earlier number. For instance, the equation $x-8=0$ gives $x=8$ and if we are having Arithmetic modulo 7 then $x=8\equiv 1 \pmod 7$.

The equation $x+5=0$ gives $x=-5\equiv 2 \pmod 7$.

Also the equation $2x=5$ gives $x=5/2$

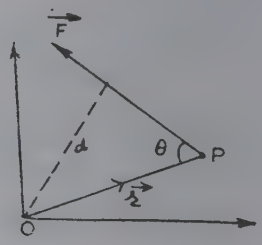
but we can write $2x=5\equiv 12 \pmod 7$ so that $x=6$.

Modulus. The absolute value of a quantity without taking into account its sign. Modulus of x is denoted by $|x|$. $|x|=x$ if $x\geqslant 0$ and $|x|=-x$ if $x\leqslant 0$, so that $|5|=5$ and $|-5|=5$. The modulus of a vector is its length or magnitude. The modulus of a complex quantity $x+iy$ is equal to $+\sqrt{x^2+y^2}$.

Moment. In a set of observations of a variable x if d represents deviation (generally from mean) then moment of nth order or nth moment is $\frac{1}{N}\sum d^n$ where N is the number of observations. Obviously, the second moment is variance and first moment is zero if the derivations are taken from mean only.

Moment of Couple. See *Couple, Torque.*

Moment of Force. The measure of turning effect of a force acting on a body, about a given point. If P is the point of application of the force, the moment of the force F acting through the point P about a given point O is the vector (cross) product of the position vector $\vec{r}$ $(=\vec{OP})$ and the force $\vec{F}$, that is equal to $\vec{r} \times \vec{F}$. Its magnitude is $Fr \sin\theta = Fd$ and direction is along the normal to a plane through OF and through F and sense determined by the right handed screw system.



Moment of Inertia. *Of a body rotating about an axis.* The moment of inertia I is given by $I=mk^2$, where m is the mass of the body and R the *radius of gyration.*

Moment of Mass. If m is the mass of a particle, d its distance from a given point, line, or plane, the moment of mass of the particle about the point, line, or plane is mx.

Momentum. If a particle is in motion, its momentum at any time t is mv where m is the mass of the particle and v its velocity at that time.

Momentum, Conservation of. See *Conservation of Momentum, Law of.*

Monomial. An expression with single term.

Monotonic. A *monotonic increasing function of a variable* x is one which does not decrease with increase in x and a

monotonic decreasing function of x is one which does not increase with increase in x.

Monotonic Sets. A system of sets in which any two sets will be such that one is always contained in the other.

Month. The moon length of time. The term is used in three ways, namely, (*i*) The *calander month*, one of the twelve divisions of the year, with 28, 29, 30, or 31 days, (*ii*) Any four consecutive weeks or any two consecutive fortnights, (*iii*) The *lunar month*, the time interval between consecutive full moons.

Multinomial. An algebraic expression containing two or more number of terms. It is known as *binomial, trinormial, quadrinomial*, etc. when it has two, three, four, etc. number of terms. See *Monomial, Polynomial.*

Multiple. An integer or expression which has a given integer or expression as a factor, is known as multiple of that factor. For example, 6 is a multiple of 2 and (x^2-5x+6) is a multiple of $(x-3)$.

Multiple Integral. If an integral is a function of more than one variable and integration is carried repeatedly, once with respect to one variable and next time with respect to another variable, and so on, over different regions. In case of two variables it is called *double integral* and in case of three variables of integration it is called *triple integral.*

Multiple Point. A point on a curve from which two or more branches of the curve pass. It may be a *double point* if two branches of the curve pass through that point, a *triple point* if three branches of the curve pass through that point, and so on, *up to n-tuple.*

For example, the curve with equation

$$y^2=ax^3$$

has a double point at origin.

Multiple Root. Syn. Repeated Root.

Multiplicand, Multiplier. In a product AB in whic say 'A is multiplied with B', B is said to be the *multiplier* an A as *multiplicand.*

Multiplication. The arithmetic operation of finding th product of two or more quantities. Multiplication of a wit b as $a.b$ is equivalent to adding a to itself b times. Th operation is commulative, *i.e.*, $ab=ba$ when we deal wit numbers. In this case the identity element is 1, so tha $a.1=a=1$. a being any number. The associativity is also vali here so that $a(bc)=(ab)c$.

Multiplication in case of vectors is different. The multipl cation of a vector $\vec{a}$ wilh a scalar k is another vector in t direction of $\vec{a}$ and magnitude k times that of $\vec{a}$. Multiplicatio of vectors $\vec{a}$ and $\vec{b}$ is defined in two ways. First is denot by $\vec{a} \cdot \vec{b}$ which results in a scalar quantity equal to

$$|\vec{a}| . |\vec{b}| . \cos\theta$$

where θ is the angle between $\vec{a}$ and $\vec{b}$. This is known as *sca* or *dot product*. Second is denoted by $\vec{a} \times \vec{b}$ which results in vector whose magnitude is equal to $|\vec{a}| . |\vec{b}| \sin\theta$ a which is in the direction normal to that of the plane containi vectors $\vec{a}$ and $\vec{b}$. The sense of direction is determined by *ri handed screw system.* It is known as *vector* or *cross product.*

Multiplication of Matrices. See *Matrix.*

Multiplier. See *Multiplicand, Multiplier.*

N

Nadir. The lower most point on the celestial sphere determined by the plumb line. The zenith, the observer, and nadir are always in a straight line. See *Zenith.*

Napierian Logarithm. See *Logarithm* to the natural base *e*

Napier's Formulae. The following formulae given by Napier are used in spherical trigonometry ; these connect the sides *a*, *b*, *c* and opposite angles α, β, γ of a spherical triangle.

$$\sin \tfrac{1}{2}(a-b)/\sin \tfrac{1}{2}(a+b)=\tan \tfrac{1}{2}(\alpha-\beta)/\tan \tfrac{1}{2}\gamma,$$

$$\cos \tfrac{1}{2}(a-b)/\cos \tfrac{1}{2}(a+b)=\tan \tfrac{1}{2}(\alpha+\beta)/\tan \tfrac{1}{2}\gamma,$$

$$\sin \tfrac{1}{2}(\alpha-\beta)/\sin \tfrac{1}{2}(\alpha+\beta)=\tan \tfrac{1}{2}(a-b)\tan \tfrac{1}{2}c,$$

$$\cos \tfrac{1}{2}(\alpha-\beta)/\cos \tfrac{1}{2}(\alpha+\beta)=\tan \tfrac{1}{2}(a+b)\tan \tfrac{1}{2}c.$$

See *Spherical Triangle.*

Nappe. Conical surface has two identical parts on both sides of the vertex and each part is called *nappe.*

Natural Logarithm. Syn. *Napierian Logarithm.*

Natural Numbers. Positive integers or whole numbers.

Nautical Mile. Unit of distance used in navigation. It is equat to 6080 feet.

Negation. See *Algebra of Propositions.*

Negative. See *Positive. Negative.*

Negative Quantity. A quantity of time, temperature, length, etc., measured from zero in a direction opposite to that chosen as positive. See *Negative, Positive.*

Neighbourhood. A small interval about a given point. For example, the neighbourhood of 0 is $(-\epsilon, \epsilon)$ where ϵ is

a small positive number, thus $(-1, 1)$, $-(\frac{1}{2}, \frac{1}{2})$, $\left(-\frac{1}{15}, \frac{1}{15}\right)$ $\left(-\frac{1}{100}, \frac{1}{100}\right)$, etc., are all neighbourhoods of 0. The neighbourhood of the point $x=a$ is taken as $(a-\epsilon, a+\epsilon)$.

Nested Intervals. A sequence of intervals in which

(*i*) each interval is included in the preceding interval,

(*ii*) length of the interval approaches to zero,

(*iii*) No two different numbers can be in all the intervals of the nest,

(*iv*) a nest of intervals represents a number if the number lies in side all the intervals of the nest.

Nested Sets. A set of sets in which one set is contained in another.

Nesting. In programming, the embedding of a subroutine or loop of instructions in another subroutine or loop, which again may be embedded within another, and so on. For example, nested *DO* loops.

Newton. A unit of force. One *Newton* is the force which produces an acceleration of one metre per second per second in a mass of one kilogram. This unit comes under *M. K. S.* system. It's symbol is *N* and is known after Sir Isaac Newton.

Newton's Law of Gravitation. Two particles attract each other with a force which is directly proportional to the product of their masses and inversely proportional to the square of the distance between them. If instead of two particles we have two bodies the distance between them is to be taken as distance between their centres of masses.

Newton's Law of Motion. (*i*) A body continues to remain at rest or to remain in motion in a straight line with uniform velocity unless acted upon by an external force (law

inertia); (*ii*) The rate of change of momentum is proportional to the force applied, the change in momentum being in the direction of the force itself; (*iii*) Every action has an equal and opposite reaction. These are the basic laws of mechanics formulated by Sir Isaac Newton.

Newton-Raphson Method. A method of finding a root of an equation by successive approximations in the form of the iterations. A rough value of root, of course, should be already known for the iterations to begin with. The algorithm or the general formula for successive approximations is

$$x_{n+1}=x_n-f(x_n)/f'(x_n)$$

where x_n is the n^{th} approximation. The formula is obtained by using Taylor's theorem. See *Iterations, Taylor's Theorem.*

Nine Point Circle. In the plane of any triangle ABC there are nine points through which one and only one circle can pass; the nine point are :

Three points being the mid-points D, E, F of the sides BC, CA, AB; three points being the feat of the altitudes G, H, I drawn from the vertices A, B, C; three points being the mid-points J, K, L of the line segments OA, OB, OC of intersection where O is the orthocentre, that is the point of concurrence the altitudes AG, BH, CI. The centre of the circle of these nine points is N which actually is the mid-point of the line segment OT where T is the circumcentre of the triangle ABC. The radius of this circle is equal to $(abc/8\Delta)$ where Δ is the area of the triangle given by $\Delta=\sqrt{s(s-a)(s-b)(s-c)}$ and $s=(a+b+c)/2$. See *Altitude, Orthocentre, Circumcentre.*

Node. A double point on a curve, if it has any, at which the two tangents to the two branches of the curve are real and distinct.

Nomogram. A graphic device consiting of three parallel lines, each one has a scale for one of the three related variables. If values of any two variables are known then that of the third

can be obtained by joining two points on the two straight lines and extending the line so that this line intersects the third line and determines the corresponding values on the third line. It is also known as *time chart*. For example, if the lines respresent temperature, volume, and pressure, then given a volume of a gas and its pressure, the temperature of the gas can be easily estimated.

Nonagon. A polygon with nine sides. A regular nonagon will have nine sides equal and hence nine equal angles at the vertices.

Non-Euclidian Geometry. Any system of geometry in which the postulates of Euclidian geometry are not valid. In Euclidian geometry we know that from a point outside a given line only one parallel line can be drawn. In non-Euclidian geometry two possibilities exist :

(*i*) Through a point outside a straight line no line parallel to the given line can be drawn.

(*ii*) There are several lines parallel to the given line. When the first possibility is satisfied, it is called a *elliptic geometry* and when the second possibility is satisfied it is called a hyperbolic geometry.

Normal. Syn. *Perpendicular*. A line or a plane is said to be normal to a curve or a surface if it is perpendicular to the tangent line or tangent plane at the point where it meets the curve or surface.

Normal Distribution. A frequency distribution in which the variation of a variable quantity x about its mean $\bar{x}$ is purely random. A normal distribution has the probability density function

$$f(x) = \frac{e^{-\frac{1}{2}\left(\frac{x-\bar{x}}{\sigma}\right)^2}}{\sigma\sqrt{2\pi}}$$

where σ is the standard deviation and σ^2 the variance. The graph of $f(x)$ against value of x is bell-shaped and symmetric

about the line $x=\bar{x}$. A normal distribution is known as standard normal distribution if $\bar{x}=0$ and $\sigma^2=1$. If we make the transformation $z=\frac{x-\bar{x}}{\sigma}$, the distribution of this new variable z will be a standard normal distribution.

Normal Equation of Line. In plane, if p is the length of the perpendicular from the origin upon a line and θ is the angle made by this perpendicular with the positive direction of x-axis then the normal equation of the line is $x \cos \theta + y \sin \theta = p$.

Null Matrix (Zero Matrix). A matrix with all elements equal to zero. See *Matrix*.

Null Sequence. A sequence with zero as its limit.

Null Set. Syn. *Empty Set.* A set having no element.

Null Vector. A vector of magnitude zero.

Number. A term used with different interpretations for counting and measuring. The earliest concept was that of counting numbers or natural numbers or whole numbers. These were used for counting giving size of collections (*cardinal numbers*) or for order (*ordinal numbers*). These were based on Hindu numerals (0, 1, 2,..., 9) which were exposed to Europe only in fourteenth century. The number 0 was used as place holder and it facilitated calculation with numbers. The operations of addtion and multiplication were freely used. Then developed the concept of directed numbers so as to get positive as well as negative integers with 0 as separator. This gave rise to the operation of subtraction. Then the concept of ratio of integers was introduced and we get rational numbers, thus, the operation of division came into being. However, some pairs of lengths could not be expressed as rational numbers and there came the concept of irrational numbers, *e.g.*, ratio of lengths of side and diagonal of a square, ratio of lengths of diameter and circumference of a circle. The real numbers are the totality of rational and irrational numbers which include surds also. Later, the transcedental numbers

and complex numbers were introduced into the fold
numbers.

Number Line. An infinitely long line on which positiv
and negative numbers can be represented by points on bo
sides of a fixed point taken as zero (conventionally the positiv
numbers on the right of zero and negative numbers on the left

Numeral. See *Numbers.*

Numerator. The number N in a fraction of the for
N/D, where D is the denominator. See *Denominator.*

Numerical Value. Syn. *Absolute Value.*

Numerical Analysis. The branch of Mathematics i
which we study and analyse the methods of approximat
computations. For example, the study of iterative method
error calculations, etc.

Numerical Integration. Study of numerical methods fo
evaluating integrals approximately, upto desired degree o
accuracy. It includes the famous methods like trapezoida
rule, Simpson's rule, etc.

O

Oblate. See *Ellipsoid.*

Oblique. Neither parallel nor perpendicular to a given
direction which is mostly the x-axis.

Oblique Coordinates. These are the cartesian coordi-
nates for which the coordinate axes are not mutually perpen-
dicular.

Obtuse Angle. An angle greater than a right angle (90° or $\pi/2$ radians) and less than a straight angle (180° or π radians). Compare *Acute Angle.*

Obtuse Triangle. A triangle with one angle as obtuse.

Octagon. A plane figure with eight sides. If all the eight sides are equal it is called a *regular octagon.*

Octahedron. A polyhedron with eight faces. A regular octahedron will have all the faces as *equilateral triangles.* See *Polyhedron.*

Octal System. A number system based on the number eight. In the octal number system we have eight digits 0, 1, 2, ..., 7 only while in decimal system we have ten digits 0, 1, 2, ..., 9 and in binany system only two 0, 1.

Octant. One of the eight regions into which space is divided by three mutually perpendicular axes of a three dimensional coordinate system. The first octant has x, y, z all positive, second octant has x negative, y and z positive, and so on.

Odd Function. A function $f(x)$ is called odd if $f(-x) = -f(x)$ for all x. For example, x^3, sin x, tan x, x^7, etc. Compare *Even Function.*

Odd Number. An integer not divisible by 2. For example, 7, 11, -5, -23 etc.

Ohm (Symbol Ω). Unit of electrical resistance. It is equal to the resistance through which a current of one ampere will pass when there is a potential difference of one volt across it.

One-One Correspondence. A mapping, correspondence, or relation between the elements of two sets such that each element of the first set is mapped into only one element of the second and vice-versa. Symbol used for this is $\sim$ and we say that $A \sim B$.

Open Curve. A curve in which the end points do not meet. For example, a parabola or a hyperbola. Compare *Closed Curve.*

Open Interval. See *Interval.*

Open Set. A set which does not include the limit points of its elements. For example, $\{x \mid 0<x<1\}$ for all real x. Here the limit points are 0 and 1 which do not belong to the set. Compare *Closed Set.*

Operator (Operation in Set). A phenomenon in a set in which two or more elements of the set combine into a single element which may or may not belong to the set. If the resulting element belongs to the set the set is said to be closed with respect to that operation. A *binary operation* involves a pair, a *ternary* operation, a triple etc. Usual operators are addition, subtraction, multiplication, division, taking square root, logarithms, etc.

Operator, Differential. In $\frac{dy}{dx}$, $\frac{d}{dx} \equiv D$ is a differential operator. $D^2 \left(\equiv \frac{d^2}{dx^2} \right)$ is a double differential operator. In $\frac{d^2y}{dx^2} + x\frac{dy}{dx} + 4y$, $(D^2 + xD + 4)$ is as differential operator operated on y.

Opposite Angles, Sides, Vertices of Polygon. Two angles, sides, vertices having equal number of sides between them round a polygon.

Opposite Angles, Sides, Vertices of Triangle. In any triangle each side is opposite a vertex or angle, and vice versa.

Orbit. The path traversed through space by a moving object. The orbits of planets and satellites are ellipses with eccentricity e quite small. The orbits of comets are elongated ellipses with eccentricity quite near 1.

Order of Derivative. See *Derivative.*

Order of Determinant. See *Determinant.*

Order of Differential Equation. See *Differential Equation.*

Order of Group. See *Group.*

Order of Matrix. See *Matrix.*

Order of Root. The number of times a root of an equation appears.

Order Relation. For a set R of rational numbers, the relation '$\leqslant$' is an order relation with the following properties (for $a, b, c \in R$).

Trichotomy. One of the relations $a<b$, $a=b$, $a>b$ for any $a, b, c \in R$, is true.

Transitivity. If $a<b$, $b<c$, then $a<c$.

Density. If $a<b$, there exists a rational number c such that $a<c<b$.

Extension. For any element a there exist two elements b and c such that $b<a<c$.

Consistency. If $a<b$, then $a+c<b+c$ and $ac<bc$ for any $c>0$.

Ordered Pair. Two elements which are to be considered only when they are written in a particular order. For example, the coordinates of a point in a plane form an ordered pair (x, y). In general the pair (y, x) will refer to a different point than (x, y).

Ordered Set. A set of elements satisfying the following properties. (*i*) For any two elements a and b, $a<b$, $a=b$, or $a>b$ (*ii*) If $a<b$ and $b<c$ then $a<c$ and if $a>b$ and $b>c$ then $a>c$.

If the above conditions are satisfied only by some of the elements of the set, the set is said to be *partially ordered set.*

Ordered Triple. Three elements which are written only in a particular order. For example, the coordinates of a point in space form an ordered triple (x, y, z).

Ordinal Number. A number which defines a place in an ordered sequence of whole numbers. Used in counting as first, second, third, fourth, etc. in a finite set.

Ordinate. The y-coordinate of a point in plane. See *Cartesian Coordinates.*

Origin. A fixed point in plane or space, from which distances are measured. It is the point of intersection of co-ordinate axes.

Orthocentre. A point which is the common point of intersection of the three altitudes of a triangle. Altitudes of a triangle are the lines from each vertex drawn perpendicular to the opposite sides. The triangle formed by joining the feet of the altitudes is called *pedal triangle.*

Orthogonal. Term referring to being right angled.

Orthogonal Basis Vectors. Mutually perpendicular basis of vectors. See *Basis Vectors.*

Orthogonal Circles. The pair of circles intersecting orthogonally, that is, tangents to the circles at the point of intersection, are at right angles to each other and each tangent passes through the centre of the other circle.

Orthogonal Projection. A geometric mapping which gives the image on a line or a plane by lines drawn perpendicular to the line or plane.

Orthogonal Trajectory. A member of a family of curves which cut every member of another family of curves orthogonally, that is, at right angles.

Orthogonal Vectors. Vector quantities which are mutually perpendicular in a plane or space.

Oscillation. A regularly repeated movement in one period of oscillation ; in other words, total movement in any vibratory motion in a time taken by a particle from one point to the same point returning in the same direction with the same velocity.

Ounce. (1) British unit of mass equal to one sixteenth of a pound. It is equivalent to 0.028349 kg.

(2) A unit of capacity called fluid ounce equal to one twentieth of a *pint*.

Output. The results and other information desired to be collected from the output device of a computer. Output is there because of feeding the data and instructions to the computer through its input device. ~~See *Input*.~~

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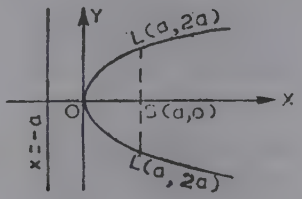
P

Paper Tape. A long strip of paper or of thin plastic is also used as input and output device in computers. Information is recorded on this tape in the form of round holes punched in rows across the tape.

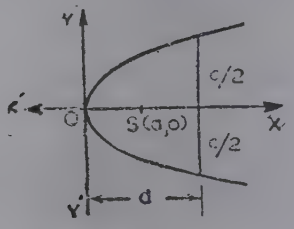
Pappus's Theorem. Two of the theorems given by Pappus are very useful. First theorem states that when an arc of a plane curve revolves about a line in its plane not cutting the curve, the surface area generated is the product of the length of this arc and the circumference of the circle traversed by its centroid. The second theorem states that the volume of a solid of revolution generated by a plane area which revolves about a line not crossing it, is the product of the area under rotation and the circumference of the circle traversed by the centroid of the plane area.

Parabola. A conic section with eccentricity 1. The standard equation of the parabola is $y^2=4\,ax$ with vertex at the origin, axis of the parabola along x axis, directrix $x=-a$, focus $(a, 0)$ and length of the latus rectum $4a$. See *Conic, Focus, Directrix.* Its parametric equations are $x=at^2$, $y=2\,at$ where t is the parameter.



Parabolic Motion. The motion of a projectile.

Parabolic Segment. The area bounded by a chord of a parabola perpendicular to its axis and the curve, cut off by the chord. The area of the parabolic segment is $2\,cd/3$ where c is length of the chord and d its distance from the vertex of the parabola.



Paraboloid. A geometric surface in which the cross section in any plane passing through a central axis is a parabola. A *Paraboloid of revolution* is obtained by revolving a parabola about its axis of symmetry. The equation $x^2/a^2+y^2/b^2=2cz$ represents an *elliptic parabolid* since its sections parallel to coordinate planes give one set of ellipses and two sets of parabolas. The equation $x^2/a^2-y^2/b^2=2\,cz$ representsa *hyperbolic parabolid* because the similar sections give one set of hyperbolas and two sets of parabolas.

Paraboloid of Revolution. See *Paraboloid.*

Paradox. An apparent contradiction between two conclusions both being supported by reasoning.

Parallel. The property of being in the same direction but remaining the same distance apart.

Parallel Curves. Points on the curves have a one-one correspondence such that the tangents at a pair of points are parallel lines, the distance between these parallel lines remain the same for all pairs of points.

Parallel Forces. Forces acting along parallel lines. Resultant of parallel forces can be obtained by adding their magnitude (considering their signs also). Two parallel forces of equal magnitude but in opposite direction and at some distance from each other form a couple which will have a turning effect.

Parallel Lines. In a plane, these are lines having no point common no matter how far these are extended.

Parallel Planes. Planes lying in space which do not have any common point no matter how far they are extended. For example, equation $2x+3y-5z+d=0$ represents parallel planes for changing values of d.

Parallelogram. A quadrilateral whose opposite sides are parallel and equal in length. In a parallelogram the opposite vertical angles are equal and diagonals bisect each other.

Parallelogram of Forces. See *Parallelogram of Vectors.*

Parallelogram of Velocities. See *Parallelogram of Vectors.*

Parallelogram of Vectors. A graphical method for finding the resultant of two vectors. If the vectors $\vec{a}$ and $\vec{b}$ acting at the point O are represented by the line segments $\vec{OA}$ and $\vec{OB}$ then their resultant $\vec{r}$ is represented by $\vec{OC}$ the dia-

gonal of the parallelogram $OACB$. $\vec{r}$ is also known as vector sum of $\vec{a}$ and $\vec{b}$. The other diagonal represents the vector difference of $\vec{a}$ and $\vec{b}$.

Parallelopiped. A solid bounded by six faces in three pairs of similar, parallel and equal parallelograms. It is called *rectangular parallelopiped* if all its faces are rectangles.

Parameter. An arbitrary quantity included in an equation gives a family of curves. For example, in the equation $y=mx+c$ where m is constant and c is arbitrary, c is taken as parameter as this represents a set of parallel lines for different values of c.

Parametric Equation. Here the cartesian coordinates x and y are expressed in terms of one or two parameters. For example, the parametric equations of the circle $x^2+y^2=a^2$ are $x=a\cos\theta$, $y=a\sin\theta$; of parabola $y^2=4ax$ these are $x=at^2$, $y=2at$; of ellipse $x^2/a^2+y^2/b^2=1$ are $x=a\cos\theta$, $y=b\sin\theta$; of hyperbola $x^2/a^2-y^2/b^2=1$, $x=a\sec\theta$, $y=b\tan\theta$, of rectanlar hyperbola $xy=c^2$, $x=ct$, $y=c/t$.

Partial Derivative. Rate of change of a function of several variables with respect to one variable, others being treated as constants. If $f(x, y)$ is a function of two independent variables x and y, $\frac{\partial f}{\partial x}$ denotes the partial derivative of f with respect to x, treating y as constant, and $\frac{\partial f}{\partial y}$ represents partial derivative of f with respect to y, treating x as constant. Thus,

$$\frac{\partial f}{\partial x} = \lim_{\delta x \to 0} \frac{f(x+\delta x, y)-f(x, y)}{\delta x},$$

$$\frac{\partial f}{\partial y} = \lim_{\delta y \to 0} \frac{f(x, y+\delta y)-f(x, y)}{\delta y}.$$

Partial Differential. If f is a function of several variables $x_1, x_2, x_3, x_4, \ldots, x_n$, the infinitesimal change δf in f resulting from the infinitesimal changes $\delta x_1, \delta x_2, \ldots, \delta x_n$ in $x_1, x_2, \ldots, x_n$ respectively, is given by

$$\delta f = \frac{\partial f}{\partial x_1}\delta x_1 + \frac{\partial f}{\partial x_2}\delta x_2 + \ldots + \frac{\partial f}{\partial x_n}\partial x_n.$$

Partial Differential Coefficient. See *Partial Derivative.*

Partial Differential Equation. An equation which contains partial derivatives of a function with respect to different variables. For example, the Laplace equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0,$$

which is encountered in studying the problems on gravitational and electro-magnetic fields.

Partial Fractions. Fractions of the form

$$\frac{A}{(ax+b)^n}, \quad \frac{Ax+B}{(ax^2+bx+c)^n}$$

where n is a positive integer. A given fraction in the proper fraction form can be decomposed into partial fractions whose denominators are factors of the denominators of given fraction.

Partial Sum. The nth partial sum of the infinite sequence $\{u_r\}$ is denoted by S_n and is given by

$$S_r = u_1 + u_2 + \ldots + u_r,$$

so that $S_1 = u_1$, $S_2 = u_2 + S_1$, $S_3 = u_3 + S_2$, etc.

Partially Ordered Set. See *Ordered Set.*

Particle. An abstract simplification of real a object. Here we think of the mass being concentrated at the *centre of mass* of the object.

Pascal's Distribution. Also known as *negative binomial distribution.* This is a distribution of the number of indepen-

dent trials performed upto rth success. The probability that the number of trials, x, is equal to k, is given by

$$P(x=k)={}^{k-1}C_{r-1}\, p^r q^{k-r}.$$

The mean and variance of this distribution are r/p and rq/p^2 respectively.

Pascal's Triangle. According to Binomial theorem

$$(1+x)^n=1+{}^nC_1x+{}^nC_2x^2+{}^nC_3x^3+\ldots+{}^nC_rx^r+\ldots+x^n.$$

If the coefficients are calculated for different values of n and arranged in the following pattern, this is known as Pascal's triangle :

$n=0$						1						
$n=1$					1		1					
$n=2$				1		2		1				
$n=3$			1		3		3		1			
$n=4$		1		4		6		4		1		
$n=5$	1		5		10		10		5		1	
$n=6$	1	6		15		20		15		6		1

and so on.

Pedal Curve. At a point P on a curve, tangent PT is drawn where T is the foot of the perpendicular from a fixed point O (generally origin) on the tangent. Then the locus of this point T is known as pedal curve. For example, the pedal curve of a parabola with respect to focus as the fixed point, is the tangent at the vertex.

Pedal Equation. At a point P on a curve, tangent is drawn. On this tangent, T is the foot of the perpendicular drawn from the pole and p is the length of the perpendicular. The radius vector of $P=OP=r$. Then a relation between p and r is called pedal equation of the curve. For example, the pedal equation of the ellipse $x^2/a^2+y^2/b^2=1$, focus taken as pole, is $b^2/p^2=2a/r-1$.

Pedal Triangle. See *Orthocentre.*

Pendulum. A body that oscillates freely under gravity. A *simple pendulum* is one which consists of a small mass oscillating to and fro at one end of a weightless cord whose other end is attached to the point of suspension. For small oscillations the motion of the mass is approximately *simple harmonic motion* with *period* of oscillation equal to $2\pi\sqrt{l/g}$ where l is the length of the cord from the point of suspension to the centroid of the mass. A *compound pendulum* is one in which a rigid body oscillates about a point. The period of oscillation here depends upon the moment of inertia of the body. For small oscillations the period is the same as that of simple pendulum except that l is to be replaced by $\sqrt{1+\frac{k^2}{h^2}}$ where k is the radius of gyration and h is the distance between the point and the centre of mass.

Pentagon. A polygon with five sides. A regular pentagon will have five equal sides and all its vertical angles will be 108°.

Percent. Symbol %. Hundredth (s).

Percentage. A common fraction with 100 as its denominator. Thus, $N/100$ may be expressed as N percent or $N\%$.

Percentage Error. Relative error multiplied by 100.

Percentile. One of the set of 100 points that divide a set of data arranged in numerical order into 100 parts. The nth percentile denoted by P_n is the value below which $n\%$ of the data lies and above which $(100-n)\%$ of data lies.

Perfect Cube. See *Perfect Power.*

Perfect Power. A number or a polynomial which is some power of a number or a polynomial respectively. Thus, 9 is a perfect square, 64 is a perfect cube, (x^2+4x+4) is a perfect square.

Perfect Square. See *Perfect Power.*

Perimeter. The length of a plane figure around its edge. For example, Perimeter of a square is four times the length of its side. Length of a closed curve is its perimeter, so that the perimeter of a circle is its circumference.

Period. The time for completing one cycle of a regularly repeated phenomenon, like oscillation, wave motion etc. It is the reciprocal of frequency.

Period of Function. See *Periodic Function.*

Periodic Curve. The graph of a *periodic function.*

Periodic Function. A function that repeats the same value at equal intervals of the variable. Mathematically, if

$$f(x+T)=f(x)$$

for all x then the function $f(x)$ is called periodic function with period T. All the trigonometric functions are periodic. See *Period.*

Periodic Motion. Any form of a regularly repeated motion is periodic motion. Oscillation of a simple pendulum is a typical example of periodic motion. See *Pendulum.*

Permutation. Number of possible arrangements in choosing r things from a set of n different things. It is denoted by ${}^nP_r$ in which P stands for the word permutation.

$${}^nP_r=\frac{n!}{(n-r)!}$$

See *Combination, Factorial.*

Perpendicular. At right angles. A line which is at right angles to any other line or plane. In other words this perpendicular intersects other line or plane normally. Symbol used for this is $\perp$. See *Normal.*

Perpendicular (Right) Bisector of Line Segment. The line at right angles and bisecting a given line segment.

Perpendicular Curves. Curves which intersect in such a way that the tangents to two curves at the point of intersection are perpendicular to each other.

Perpendicular Line to Plane. A line at right angles to any two lines in the plane. Such a line is also known as normal to the plane.

Phase. A stage in a cycle in any periodic system at any particular time. Two periodic waves are said to be *in phase* if the maxima and minima of one, coincide with the maxima and minima of the other.

Phase Difference. If a simple wave has the equation

$$y=a \sin (ft-x/\lambda)2\pi$$

the phase of this wave will be $2\pi(ft-x/\lambda)$ and the phase difference between two points $x=x_1$, $x=x_2$ at the same time t is equal to $2\pi(x_1-x_2)/\lambda$. See *Phase*.

Pi, π. The ratio of the circumference of any circle to its diameter is always a constant denoted by π. It is a transcedental as well as an irrational number whose true value is 3.1415926536... and can be taken approximately as 3.1416. It was introduced by Archimedes who has shown that the exact value of π lies between $\frac{220}{70}$ and $\frac{223}{71}$.

Pictograph. A diagram which represents statistical data in pictorial form.

Pie Chart. A diagram in which proportions are illustrated by sectors of a circle. For examples the tax collections from different sources is usually represented in proportion by a pie chart.

Place Value. The place value of a digit in a number is the digit multiplied by 10^0, 10^1, 10^2, 10^3, etc. according as the digit appears in the number as ones, tens, hundreds, thousands etc. respectively.

Plane. A flat surface which contains wholly the straight line drawn through any two points taken on the surface. Its general equation is $ax+by+cz+d=0$ where a, b, c are the direction ratios of the normal to the plane.

Plane Co-ordinates. Cartesian or polar co-ordinates in two dimension which determine the position of a point in a plane.

Plane Figure. A geometric figure which lies in a plane.

Plane Geometry. Study of geometric figures in a plane.

Plotter. An ouput device of a computer, which plots the results of a program by drawing lines and curves on a plane paper. The device may even contain two or more pens with different colours of ink for drawing graphs.

Plotting. The process of marking points for drawing a graph.

Plumb-Line. A vertical line determined by the position assumed by a string rigidly supported and carrying a plumb-bob.

Plus (Symbol +). Symbol is used for summing or addition.

Point. That which has position but no magnitude.

Point (Decimal). See *Decimal.*

Point of Contact. The point common to a curve and a tangent to the curve.

Point of Inflexion. A point on a curve such that on one side of the point the curve is concave and on the other side it is convex. More precisely, approaching from one side of the point of inflexion the slope of the tangent to the curve increases while moving away from it on the other side, the slope decreases. For example, the curve $y=x^4-4x^3$ has $x=2$ as a point of inflexion, $\frac{d^2y}{dx^2}=12x^2-24x$ which vanishes at $x=2$ and changes its sign at $x=2$.

Poisson's Distribution. A probability distribution of a discrete random variable x. For a variable (r) which can take values in the range 0, 1, 2,.....and has a mean value μ as $P(r)=\frac{\mu^r}{r!}e^{-\mu}$. A binomial distributionwith a small frequency

of success in a large number n of trials can be approximated by a Poisson's distribution with mean np.

Poisson's Ratio. In elasticity, the ratio of transverse strain to the longitudinal strain, when the elastic body is subjected to longitudinal stress.

Polar Coordinates. A system of co-ordinates for defining position of a point P in terms of its distance OP from a fixed point O and the angle which OP makes with the initial line OX, point O being pole. OP is called *radius vector* and the angle XOP taken as θ measured anticlockwise. The angle θ is called as *argument* or *amplitude*. To convert a cartesian equation into polar we make the transformation

$$x=r\cos\theta, \qquad y=r\sin\theta$$

in which $r=\sqrt{x^2+y^2}$, $\theta=\tan^{-1}(y/x)$.

Pole. (1) See *Polar Co-ordinates*, (2) One of the points of intersection on the earth's surface by its axis of rotation, (3) See *Pole and Polar of Conic*.

Pole and Polar of Conic. If a variable secant is drawn from a given point P to a conic meeting the conic in points A and B. On AB find a point Q which is such that P and Q are harmonic conjugates of A and B, that is,

$$PA/PB=QA/QB,$$

then the locus of Q is called the polar of P and P is called pole This polar actually comes out to be the chord of contact CD of the point P. On CD there would be a point T such that AB is the chord of contact of T.

Polygon. A plane geometric figure consisting of a number of sides as line segments. The polygon will be called *regular* if all its sides are equal and hence all its angles are equal.

Polyhedron. A solid figure bounded by a number of faces each of which is a polygon. The point of intersection of three or more faces is called a *vertex* of the polyhedron, the line of intersection of two plane faces is called an *edge* of the polyhedron. A *regular* polyhedron is one in which all the faces are

congruent regular polygons. There are only five regu polyhedrons with number of faces as 4, 6, 8, 12, and 20.

Polynomial. An expression of the form

$$a_0x^n+a_1x^{n-1}+a_2x^{n-2}+\ldots\ldots+a_{n-1}x+a_n$$

where n is the degree of the polynomial. It is linear, quadrat cubic, quartic, etc. when n is 1, 2, 3, 4, etc.

Population. The whole set of items with some comm characteristic. From the population, samples are drawn carrying out statistical analysis.

Position Vector. The vector represented by aline segme from the origin to a point in space is called position vector the point under consideration. Thus, the position vector of t point (x,y,z) is $\hat{x}i+\hat{y}j+\hat{z}k$ where i, j, and k are the unit vect along the three co-ordinate axes of x,y, and z.

Positive Number. Any number that is greater than ze Such numbers are used, apart from other things, in counti things and measuring different sizes.

Postulate. See *Axioms.*

Potential. The potential at a given point is the work do in bringing a unit quantity from infinity to that point against conservative field. See *Conservative Forces.*

Potential Energy. The energy, posssessed by a bo because of its position only. If a body of mass m is put at so height h above the ground, it would now possess gravitiio potential energy equal to mgh where g is the acceleration due gravity.

Pound. A British unit of mass, equal to 0.45359237 Kg.

Pound Weight. The weight of one pound mass.

Poundal. British unit of force in F.P.S. system, equal 0.138255 newton.

Power. (1) Rate of doing work. In F.P.S. system it is measured in foot pounds per second and in C.G.S. in system centimetre-grams per second. In British unit, 1 horse power (hp) is 550 *ft-lb*/sec. (2) In algebra, it is the number of times a quantity is multiplied by itself. For example,

$$3^5 = 3 \times 3 \times 3 \times 3 \times 3,$$

here the power of 3 is 5 and in x^n the power of x is n.

Power Series. A series in which the terms are regularly increasing powers of a variable. It is of the form

$$a_0 + a_1x + a_2x^2 + a_3x^3 + \ldots\ldots + a_nx^n + \ldots$$

where a_0, a_1, a_2,.....are constants.

Precision. The number of figures in a given number. For example, a number 1.4142 is given to a precision of five figures or has a precision upto four places of decimal.

Premise. A preposition which is assumed to be true in the beginning and on which some more arguments are based.

Premium. The difference between the market value of the share and its face value, The share is said to be *above par*, *at par*, or *below par* according as the market value of the share is more, equal, or less than its face value respectively.

Pressure. The force acting normally on a unit area of the surface of a body and can be written as the ratio of force per unit area.

Prime Factor. Factor of a given quantity, which is a prime number. See *Prime Numbers*.

Prime Number. Any integer like 2, 3, 5, 7, 11, 13, etc. which does not have any factor other than 1 and itself. Euclid has shown that there are infinite number of prime numbers.

Prime Polynomial. A polynomial expression which does not have any polynomial factor other than constants and itself. For example, $x+y$.

Prime as Symbol. Symbols ′ and ″ and ‴ etc. placed at the top right of another symbol. Thus, (x', y') is a point different from (x, y) and (x'', y'') is a point different from (x', y').

The symbol is also used in derivatives. For example, a' represents first derivative and a'' represents second derivative of a.

Principal. A sum of money which is given to some on on interest or is borrowed from some one on interest. Se *Simple Interest.*

Principal Diagonal. Elements of a determinant or a square matrix lying in a diagonal drawn from top left to botton right. It is also known as *Leading Diaganal.*

Principle of Mathematical Inductlon. See *Induction.*

Principle of Undetermined Coefficients. If two poly nomials in one variable are equal, the coefficients of the term of same power in the two polynomials must be equal. Thi principle is very much used in finding the numeraters in partia fractions.

Principle of Moments. The principle that when a body is in equilibrium the sum of the moments of the forces in one direction must be equal to the sum of the moments of the force in opposite direction.

Printout. The computer output in the form of a numbe of pages printed by the line printer on a continuous sheet.

Prism. A solid, usually made of a glass, having congruen parallel faces (called bases) and a set of rectangles (called latera faces) formed by joining corresponding vertices of the bases.

Probability. Numerical measure of the likelihood of an event to occur. If in an observation there are n possible ways exhaustive and mutually exclusive, and, out of them in m ways the event A occurs then the probability of occurrence of the event A is given by $P(A)=m/n$. If in a random sequence of N trials of an event, M are favourable to the event, then the probability of that event occurring, is the limit of the ratio M/N when N is very very large. This lies between 0 and 1. $P(A)=0$ means the event cannot take place, $P(A)=1$ means the event is bound to occur. The events $A_1, A_2, \ldots\ldots, A_n$ are said to be *mutually exclusive* if $P(A_i \cap A_j)=0$ for $i=1, 2, \ldots, n$; $j=1, 2, \ldots, n$; $i \neq j$, and the events are said to be *exhaustive* if

$$P(A_1)+(A_2)+\ldots\ldots+P(A_n)=1. \quad \text{If } A \cap B=0$$

then $P(A \cup B) = P(A) + P(B)$.

and if $A \cap B \neq 0$ then $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

Conditional Probability is the probability of the occurence of an event A subject to the condition that B occurs, and is given by

$$P(A/B) = P(A \cap B)/P(B).$$

If A and B are *independent* then

$$P(A/B) = P(A) \text{ and } P(A \cap B) = P(A)P(B).$$

Probability Density Function. See *Random Variable.*

Product. Generally, the result of multiplication of two or or more numbers, vectors, matrices etc. For product of two sets, see *Intersection, Cartesian Product.*

Program. A set of instructions to the computer, put in a sequential order, written in a programming language, special language understandable by the computer. The instructions are written in the binary number system. Instructions along with the relevant data on which the instructions operate are fed into the computer as input. The code which includes only binary numbers 0 and 1 is called *binary codr or machine code.* The programming language which is in machine code, is called *machine language.* It is quite tedius to write program in machine code, hence the programs are written in *problem-oriented languages* or *high level languages* like FORTRAN, COBOL, BASIC, etc. and are called *source programs* which are translated by the complier into machine language. *Machine oriented languages* or *Assembly languages are* all *low-level languages. Assembler* converts assembly language programs into machine code.

Progression. A sequence of values in which each term is obtained from the preceding term in the same way. See Arithmetic Progression (A.P.), Geometric Progression (G.P.), and Harmonic Progression (H.P).

Projectile. An object projected in space with a velocity at an angle to the horizontal line through the point of projection.

The horizontal and vertical components of velocity are to be treated separately. The path trasversed by the projectile is always a parabola.

Projection. A geometrical mapping which maps a line, curve, or any shape into another as per the given geometrical rule. The phenomenon of orthogonal projection is of utmost use.

Projective Geometry. The study of how the geometric properties of one configuration, are changed when subjected to projection by some geometric rules, to another configuration. *Central Projection* are those under which the properties of the configuration subjected to such projection, remain unchanged. See *Projection.*

Prolate. See *Ellipsoid.*

Proof. A logical argument that establishes the validity of a proposition or mathematical formula. The proof comprises of a set of axioms and premises, sequentially to arrive at a formula to be proved in the form of a conclusion.

Proper Fraction. See *Fraction.*

Proportion. A relation giving the equality of two ratios in the form $a/b=c/d$. Earlier it was written as $a : b :: c : d$ which has become obsolete these days. The operation of *componendo*, *dividendo*, and *convertendo* applied to the proportion $a/b=c/d$ give

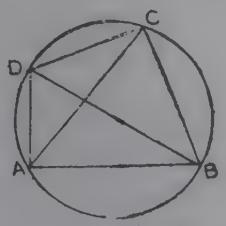
$$\frac{a\pm b}{b}=\frac{c\pm d}{d},\quad \frac{a}{a\pm b}=\frac{c}{c\pm d},\quad \frac{a}{b}=\frac{c}{d}=\frac{a\pm c}{b\pm d}$$

This is also known as *simple proportion.*

Proposition. Syn. *Theorem.* A sentence or formula in logic. See *Algebra of Proposition.*

Protractor. An instrument used in geometry and in drawing, for measuring angles. It is semi-circular in shape and has 0° to 180° marked on it.

Ptolemy's Theorem. This establishes relation between the lengths of the sides and diagonals of a cyclic quadrilateral $ABCD$: $AB.CD+BC.AD=AC.BD$.



Pulley. A machine in which we transfer power through the tension in a string wound over one or more wheels, with the help of pulleys we get more output by applying less effort.

Punched Card. See *Card.*

Pure Mathematics. Whole of mathematics can be broadly classified into two categories : Pure Mathematics and Applied Mathematics. Pure Mathematics deals with the mathematical theory and structures without intending to apply it to any field. Pure mathematics is abstract in nature.

Pyramid. It is a solid with base as a polygon and other faces are triangles with a common vertex. If the base is a regular polygon, a line from the vertex to the centre of the base is called *axis* of the pyramid. The pyramid is a *right-pyramid* if its axis is perpendicular to its base, otherwise oblique. In a regular pyramid all the lateral faces are congruent isosceles triangles making the same angle at the base. A square pyramid has a square base and four congruent triangular faces. The volume of a pyramid is one-third the area of the base multiplied by its height.

Pythagorus Theorem. In any right angled triangle the square on the hypotenuse is equal to the sum of the squares on the other sides.

Q

Quadrangle. A *simple quadrangle* is a plane geometrical figure consisting of four vertices A, B, C, D, and their joins AB, BC, CD, DA. If these line-segments do not intersect, the figure is called a quadrilateral. A complete quadrangle is a plane geometrical figure consisting of four points and six lines AB, BC, CD, DA, AC, BD. No three of the points A, B, C, D are collinear.

Quadrant. (1) A quarter of a plane. In Cartesian co-ordinate system, the first quadrant is the area in which both x and y are positive, the second the area in which y is positive and x negative, the third the area in which both x and y are negative, the fourth the area in which x is positive and y is negative. In polar co-ordinates the first, second, third, and fourth quadrants are the areas when θ the vertical angle varies from $0°$ to $\pi/2$, from $\pi/2$ to π, from π to $3\pi/2$, and from $3\pi/2$ to 2π respectively. (2) The part of a circle bounded by two perpendicular radii and a quarter of the circumference. (3) A unit of plane angle equal to $90°$ or $\pi/2$ radians.

Quadrantal Angles. Any of the angles $n\pi/2$

$$(n=0, 1, 2, \ldots\ldots).$$

Quadratic Equation. The second degree algebraic equation of the form $ax^2+bx+c=0$, where a, b, c are constants. The roots of this equation are given by

$$x=(-b\pm\sqrt{b^2-4ac})/(2a)$$

which will be real or complex according as the discriminant (b^2-4ac) is positive (including zero) or negative.

Quadratic Expression. An expression of the form ax^2+bx+c which is an algebraic expression of degree two.

Quadrilateral. A plane figure with four straight sides. Its six special cases are as follows ;

(*i*) *Trapezium* (*or Trapezoid*). Having two of its sides as parallel.

(*ii*) *Parallelogram*. Having both the pairs of its opposite sides as parallel.

(*iii*) *Rectangle*. Having with all the vertical angles as right angles.

(*iv*) *Kite*. Having two pairs of equal adjacent sides.

(*v*) *Rhombus*. Having all the four sides equal. It can also be taken as a parallelogram with equal adjacent sides.

(*vi*) *Square*. Having all the four sides equal and all the four vertical angles right angle. In can also be taken as a rectangle with equal adjacent sides.

Quadrinomial. See *Multinomial*.

Quarter. Denotes one fourth. (1) A common fraction with 4 as denominator. (2) One of the phases of moon. (3) One fourth of a year, equal to three months.

Quartic. An algebraic equation of degree four with general form as $a_0x^4+a_1x^3+a_2x^2+a_3x+a_4=0$. It is also known as *biquadratic equation*. The algebraic expression of fourth degree, of the form $a_0x^4+a_1x^3+a_2x^2+a_3x+a_4$ is called *quaratic expression* or *biquadratic expression*.

Quartic Symmetry. Symmetry in a plane figure having four axes of symmetry.

Quartile. One of the three points that divide a set of data into four equal parts, in which data has already been arranged in numerical order. The three quartiles are denoted by Q_1, Q_2, and Q_3. Q_1 is 25th percentile P_{25}, Q_2 is 50th percentile P_{50} (median), and Q_3 is 75th percentile P_{75}. See *Median*, *Percentile*.

Quinary. Associated with five.

Quintal. A term used for 100 lb or 100 kg.

Quintic Equation. An algebraic equation of 5th degree of the form $a_0x^5+a_1x^4+a_2x^3+a_3x^2+a_4x+a_5=0$.

Quintic Expression. An algebraic expression of degree five of the form $a_0x^5+a_1x^4+a_2x^3+a_3x^2+a_4x+a_5$.

Quotient. The result of dividing one number by another, in the process there may or may not be any remainder. For example, 7/5 has 1 as quotient and 2 as remainder.

Quotient Field. The set of rational numbers is a field and, as any rational number is in the form p/q where p and q are integers and $q\neq0$, the field of rational numbers is often called quotient field. See *Field*.

R

Radian. Unit of measuring a plane angle. Its size is defined by the angle subtended at the centre of a circle by an arc equal in length to the radius of the circle. $180°=\pi$ radians.

Radical Axis. The locus of a point from which two pairs of tangents of equal length can be drawn to two given circles. If $S_1\equiv0$, $S_2\equiv0$ represent the equations of two circles then $S_1-S_2=0$ will give the equation of the radical axis. Thus for the circles

$$s_1\equiv x^2+y^2+2g_1x+2f_1y+c_1=0$$

and $$s_2\equiv x^2+y^2+2g_2x+2f_2y+c_2=0$$

the radical axis is

$$2(g_1-g_2)x+2\,(f_1-f_2)\,y+c_1-c_2=0.$$

If the two circles are intersecting the radical axis will be their common secant, and if they touch each other the common tangent is the radical axis.

Radical Expression. An indicated root of a quantity. For example, $2\sqrt{3}$ or $\sqrt{3}$, $\sqrt[3]{y}$, $(=y^{1/3})$, etc. $\sqrt{}$ is the *radical sign* and $\sqrt[2]{}$ represents square root, $\sqrt[3]{}$ represents cube root, $\sqrt[n]{}$ represents the n^{th} root or root with *index n*. In $\sqrt[n]{x}$, x is called *radicand* or *base*.

Radical Sign. See *Radical Expression.*

Radicand. See *Radical Expression.*

Radius. The distance from the centre to any point on the circumference is the radius of a circle whereas the distance from the centre of a sphere to any point on its surface is the radius of the sphere.

Radius of Curvature. See *Curvature.*

Radius of Gyration. If $m_1, m_2, m_3, \ldots\ldots$ are the masses of particles of which a body of total mass M is comprised of and $r_1, r_2, r_3, \ldots\ldots$ are the distances of these particles from some axis then the moment of inertia I of the body about this axis is given by $I = MK^2 = m_1r_1^2 + m_2r_2^2 + m_3r_3^2 + \ldots\ldots$. The distance K is known as radius of gyration of the body.

Radius Vector. See *Polar Coordinates, Spherical Coordinates.*

Random. Abrupt, unordered.

Random Number Table. A table consisting of a sequence of randomly chosen numbers. The Tipet's random number table is very frequently being used particularly in random sampling. See *Sampling.*

Random Sampling. See *Sampling.*

Random Variable. (*Stochastic Variable*). A quantity that can accept any one of a number of the predicted values of the variable. A *discrete random variable X* has a definite set of possible values $x_1, x_2, x_3, \ldots\ldots, x_n$ with probabitities $p_1, p_2, \ldots\ldots, p_n$ attached to the values of the variable. Here $p_1 + p_2 + p_3 + \ldots\ldots + p_n = 1$. On the other hand, if X is a *continuous*

random variable it can take any value in a given range. The probability of x having a particular value is called the *probability density function* $f(x)$ since if $x \in [a, b]$ then $\int_a^b f(x)\,dx = 1$, that is, the area under the curve given by the probability density function $f(x)$ is always 1.

Range. (1) In statistics, the difference between the largest and the smallest values in a data. (2) In algebra, the range of a function $f(x)$ is the set of values of $f(x)$ for all possible values of x.

Rank. In statistics, the position of an object when the set of objects are orderded according to their importance or magnitude. For example, after an examination is held the students of a class can be put in order of their ranks as first, second, third, etc.

Ratio. A relation between two quantities a and b of the same variety (expressed in same units) as fraction of the form a/b.

Ratio Tests. Devices for determining the convergence or divergence of an infinite series. Cauchy's test is the ordinary ratio test while *D*' Alembert's test is the generalized one.

Rational Expression. An algebraic expression in which variables do not occur with fractional indices. For example $3x^2+4x+1$ is rational while $(3x^2+2)^{1/3}$ is not rational. It is an irrational expression.

Rational Function. A function which appears or can be expressed as quotient of two polynomials.

Rational Number. A number which can be expressed as ratio of two integers, *i.e.*, in the form p/q where p and q are integers and $q \neq 0$. 1/3 is a rational number while $\sqrt{2}$ is not and hence known as irrational number.

Reaction. See *Newton's Laws of Motion.*

Reader. A device in computer to read the data and other instructions supplied in any programming language. For example, the punched card reader, paper tape reader, magnetic tape reader etc.

Real Axis. An axis on which only real numbers can be represented. See *Real Numbers.*

Real Numbers. The set of all rational as well as irrational numbers.

Real Variable. A variable which can assume only real values in the form of real numbers.

See *Real Numbers.*

Reciprocal. Reciprocal of a quantity is equal to 1 divided by that quantity. For example, reciprocal of 5 is 1/5, of 2.3 is 1/2.3, of $(x+1)$ is $1/(x+1)$.

Rectangle. A plane figure in the form of a quadrilateral with four right angles. Its area is the product of the lengths of its adjacent sides. It becomes a square if its adjacent sides are of equal length.

Rectangular Coordinates. Cartesian coordinates with axes of coordinates mutually perpendicular.

Rectangular Hyperbola. See *Hyperbola.*

Rectangular Parallelopiped. See *Parallelopiped.*

Rectification. The process of determining the length of the arc of a curve. If the curve has the equation in cartesian form, the length of the curve from $x=a$ to $x=b$ is equal to

$$\int_a^b \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$$

Rectilinear Figure. A plane figure bounded by line segments. For example, Polygons.

Rectilinear Motion. Motion along a straight line.

Recurring Decimal. See *Decimal.*

Reduction Formulae in Integration. Formulae which reduce a given integral into a simple form. For example, the reduction formulae

$$\int \sin^n x\,dx = -\frac{1}{n}\sin^{n-1} x\cos x + \frac{n-1}{n}\int \sin^{n-2} x\,dx,$$

$$\int \cos^n x\,dx = \frac{1}{n}\cos^{n-1} x\sin x + \frac{n-1}{n}\int \cos^{n-2} x\,dx$$

$$\int \sin^p x\cos^q x\,dx = -\frac{1}{p+q}\sin^{p-1} x\cos^{q+1} x + \frac{p-1}{p+q}\int \sin^{p-2} x\cos^q x\,dx,$$

$$\int \sin^p x\cos^q x\,dx = \frac{1}{p+q}\sin^{p+1} x\cos^{q-1} x - \frac{q-1}{p+q}\int \sin^p x\cos^{q-2} x\,dx.$$

Reduction Formulae in Trigonometry. Formumlae or rather the identities which reduce *a* circular function of any angle in terms of acute angle. For example,

$\sin(90°\pm\theta)=\cos\theta$, $\sin(180°-\theta)=\sin\theta$,

$\sin(180°+\theta)=-\sin\theta$, $\sin(360°-\theta)=-\sin\theta$,

$\cos(90°-\theta)=\sin\theta$, $\cos(90°+\theta)=-\sin\theta$,

$\cos(180°-\theta)=-\cos\theta$, $\cos(180°+\theta)=-\cos\theta$,

$\cos(360°-\theta)=\cos\theta$, $\tan(90°-\theta)=\cot\theta$,

$\tan(90°+\theta)=-\cot\theta$, $\tan(180°-\theta)=-\tan\theta$,

$\tan(180°+\theta)=\tan\theta$, $\tan(360°-\theta)=-\tan\theta$, etc.

Reflection. A transformation in geometry in which any point on one side of a plane, line, or point, is mapped to a symmetrical point on the other side. Thus, reflection is defined

about a point, a line, or a plane, called *point of symmetry*, *line of symmetry* is called *axis of reflection*, and plane of symmetry is called *plane of reflection*.

Reflex Angle. An angle greater than 180° (π radians) but less than 360° (2π radians).

Reflex Relation. A relation which is such that any element x, bears the relation to itself. For example, the relation of equality in the set of numbers, $x=x$.

Regression Lines. A line $y=ax+b$ is called the line of regression or regression line of y on x if it gives the expected value of y for a given value of a random variable x. Similarly, another line $x=cy+d$ is called the regression line of x on y if it gives the expected value of x for a given value of the random variable y. Geometrically, we draw the scatter diagram of data points $(x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)$ and in that the regression lines can be drawn as the lines of best fit. See *Line of Best Fit*.

Regular Polygon. A polygon with equal sides and hence equal angles at all the vertices.

Regular Polyhedron. A polyhedron with faces as regular polygons. See *Regular Polygon*.

Relation. The relation R, from a set X to a set Y, is a proper subset of X. Y the cartesian product $\{(x, y)/x \in X, y \in Y\}$ such that xRy. See *Cartesian Product*.

Relative Acceleration, Velocity. The acceleration, velocity of a moving object with respect to another object.

Relative Density. Syn. *Specific Gravity*. The ratio of the density of a substance to that of water at 4°C in case of liquids and solids, and to that of hydrogen at 0°C at atmospheric pressure 760 mm in cases of gases.

Relative Error. For a set of measurements of the same thing the ratio of the error to the mean of measurements. On multiplication by 100 it becomes *percentage error*. See *Error*.

Relative Velocity. See *Relative Acceleration, Velocity*.

Relativity. The theory of relativity was developed by Albert Einstein. In nut shell we can say that all the natural processes are functions of time as well and time should be taken as fourth dimension. According to his theory there is nothing like *absolute value* of speed, it is always *relative*, relative to that of the observer.

Remainder. See *Dividend.*

Remainder Theorem. The theorem is used in determining the remainder in division when a polynomical $f(x)$ is divided by $(x-a)$. This gives

$$f(x)=(x-a)\,g(x)+f(a)$$

where $g(x)$ is the quotient and $f(a)$ is the remainder. Thus the remainder can be obtained by simply putting $x=a$ in the polynomial. For example, if x^3+5x^2-3x+4 is divided by $x-2$ the remainder is

$$f(2)=2^3+5.2^2-3.2+4=26$$

The theorem is also used for determining factor of a polynomial expression, since if $f(a)=0$, $(x-a)$ is a factor of $f(x)$.

Repeated Root. If $f(x)$ is divisible by $(x-a)^n$, then the equation $f(x)=0$ will have $x=a$ as a root repeated n times. For example, in the equation $x^5+x^4-2x^3-2x^2+x+1=0$, -1 is a triple root and $+1$ is a double root.

Repeating Decimal. Syn. *Recurring Decimal.* See *Decimal.*

Representative Fraction. A proper fraction indicating the scale of a map or a plan. It is the ratio of the distance on the map to the distance on the ground, both being expressed in the same units. For example, the representative fraction 1/1,000,000, represents the scale 1 cm $\simeq$ 1,000,000 c.m., i.e., 1 cm on the map represents 10 km. on the ground, thus, in the representative fraction the numerator denotes the distance on the map and th. denominator the distance on the ground, both in the same units. Further, the representative fraction 1/63,360 indicates the scale 1 inch=1 mile.

Residue Class. In modulo (modular) arithmetic, all the positive integers can be partitioned into a number of residue classes, decided by the remainders after dividing the numbers by a given positive integer. For example, in modulo (3) arithmetic, one residue class consists of the numbers 1, 4, 7, 10, 13, 16, ..., because on division each number leaves a remainder 1. A second residue class consists of 2, 5, 8, 11, 14, . . The number of such residue classes is infinite. This residue class is denoted by *residue class mod. 3.*

Resolution of Forces. See *Resolution of Vectors.*

Resolution of Vector. The process of determination of the components of a vector in two mutually perpendicular directions. These components are called the resolved parts of the given vector. These component vectors when combined give the original vector, known as resultant vector. Sometimes the term is used to denote the process of resolving the given vector in the components in oblique directions. See *Parallelogram of Vectors.*

Restitution Coefficient. The ratio of the relative velocities of two colliding bodies immediately after collision to their relative velocities before impact. If u_1 and u_2 are the velocities of the two bodies before collision and v_1, v_2 their respective velocitive after the impact, then the coefficient of restituion (e), is given by

$$e = \frac{v_1 - v_2}{u_1 - u_2}.$$

In general $e < 1$.

A perfect elastic collision conserves the kinetic energy, *i.e.*, in such a case $e = 1$

Rest Mass (m_0). The mass of an object at rest with respect to the observer.

Resultant. A vector having the same effect on an object as that of a number of vectors acting on that object. See *Resolution of Vectors.*

Retardation. Negative acceleration. See *Acceleration.*

Reverse Order. The terms of a sequence being writte from the end.

Revolution. A complete rotation about a point or a lin The revolution is complete if angle of rotation is 360°.

Rhombehedron. A polyhedron bounded by six faces, eac face being a rhombus. See *Rhombus.*

Rhombus. A plane figure bounded by four straight lin of equal lengths opposite sides being parallel. The diagona of rhombus are perpendicular bisectors of each other. The are of rhombus is $\frac{d_1 \times d_2}{2}$, where d_1, d_2 are the lengths of i diagonals.

Riemann Integral. See *Definite Integral.*

Right Angle. The angle between a horizontal plane and vertical line. It is 90° or $\frac{\pi}{2}$ radians.

Right Angled Triangle. A triangle with one ang 90° and other two acute angles. The side facing the angle 9 is called *hypotenuse.* If a and b are the sides of a right angle triangle and c the hypotenuse, then $c^2 = a^2 + b^2$.

Right Circular Cone. Syn. *Right Cone.* A cone havin a circular base and axis perpendicular to the base.

Right Circular Cylinder. Syn. *Right Cylinder.* cylinder whose base is a circle and axis perpendicular to th base.

Right hand Rule. Consider two vectors $\vec{A}$ and $\vec{B}$. Strec the right hand in such a way that the fingers point in th direction of the vector $\vec{A}$. Rotate the hand in such way tha the fingers now are set in the direction of the vector $\vec{B}$, the

the thumb indicates the direction of vector $\vec{C}$, where $\vec{A}\times\vec{B} = |\vec{A}|\,|\vec{B}|\,\sin\theta\,\hat{\eta}$. Here θ is the angle between the vector $\vec{A}$ and $\vec{B}$ and $\hat{\eta}$ the unit vector in the direction of the product vector $\vec{C}$.

Right Handed Screw System. In the vector multiplication of two vectors $\vec{A}\times\vec{B}$, the vector $\vec{A}$ is thought of being moved towards the vector $\vec{B}$ and we think of a screw being moved in such a pattern. The direction of the ultimate motion of the screw determines the direction of the vector $\vec{A}\times\vec{B}$ The screw, when rotated anticlockwise, moves upward and when rotated clockwise moves downward. The principle explains the fact, $\vec{A}\times\vec{B} = -\vec{B}\times\vec{A}$.

Right Line. Syn. *Straight Line.*

Right Prism. See *Prism.*

Rigid Body. A body which undergoes negligible distortions in its shape by the application of limited force on it.

Rigidity Modulus. See *Elasticity.*

Ring. A non-empty set $R=\{a, b, c, \ldots\}$ together with two binary operations $\oplus$ and $\otimes$ defined on its elements such that $\forall\ a, b, c \in R$

1. The operation $\oplus$ is associative in R : $a \oplus (b \oplus c) = (a \oplus b) \oplus c$.

2. The operation $\oplus$ is Commutative in R : $a \oplus a = b \oplus a$

3. These exists an identity element $0 \in R$ with respect to $\oplus$: $a \oplus 0 = 0 \oplus a = a$,

4. These exists inverse $-a \in R$ for every element $a \in R$ such that $a \oplus (-a) = (-a) \oplus a = 0$.

5. The operation $\otimes$ is associative in R : $a \otimes (b \otimes c) = (a \otimes b) \otimes c$.

6. The operation $\otimes$ is distributive with respect to the operation $\oplus$ in R : $a \otimes (b \oplus c) = (a \otimes b) \oplus (a \otimes c)$

and $(b \oplus c) \otimes a = (b \otimes a) \oplus (c \otimes a)$.

The ring defined above is denoted by $\{R, \oplus, \otimes\}$.

Ring, Commutatative. A ring having the property $a \otimes b = b \otimes a$. See *Ring*.

Ring with Unity. A ring having an identity element (usually denoted by 1) with respect to the operation $\otimes$. A ring with unity is called a *Division Ring* if $\forall\ a \in R\ (a \neq 0)$, there exists $a^{-1} \in R$, such that $a \otimes a^{-1} = a^{-1} \otimes a = 1$. See *Ring*.

Rolle's Theorem. If a function $f(x)$ is continuous in the closed interval $[a, b]$ and differentiable in the open interval $]a, b[$ and $f(a) = f(b) = 0$, then there exists a points $c \in]a, b[$ such that $f'(c) = 0$. Geometrically, if a continuous curve crosses the x-axis at two points $x = a$ and $x = b$, and has a tangent at every point between $x = a$ and $x = b$ then there must be at least one point between a and b at which the tangent to the curve is parallel to the x-axis, *i.e.*, the function $f(x)$ has a maximum or minimum value between $f(a)$ and $f(b)$.

Root Mean Square Values. Syn. *R. M. S Value.* For discrete data $x_1, x_2, \ldots, x_n$

$$R.\ M.\ S.\ \text{Value} = \sqrt{\frac{1}{n} \sum_{r=1}^{n} x_r^2}.$$

For the continuous data $f(x)$ over (a, b),

$$R.\ M.\ S.\ \text{Value} = \sqrt{\frac{1}{b-a} \int_a^b [f(x)]^2\, dx}.$$

Root of Complex Number. If $z=r(\cos\theta+i\sin\theta)$, then $z^{1/n}=r^{1/n}\left[\cos\left(\frac{2k\pi+\theta}{n}\right)+i\sin\left(\frac{2k\pi+\theta}{n}\right)\right]$, is called nth root of z, where $k=0, 1, 2, \ldots, n-1$ give the n roots of z.

Root of an Equation. The value (*values*) of x satisfying the equation $f(x)=0$ is (are) called the root (roots) of the equation. In case of algebraic equation the number of roots equals the degree of the equation.

Root of a Real Number. The solution of the equation $x^n=a$, is the nth root of the real number a. If $n=2$, the root is called the square root of a (denoted by $\sqrt{a}$), if $n=3$, it is cube root of a and if $n=4$, the fourth root and so on.

Rose. A curve whose polar equation is $r=a\sin n\theta$ or $r=a\cos n\theta$, where a is a real constant and n an integer. If n is odd the curve has n loops and when n is even the curve has $2n$ loops. The curves are known as n−leaved or $2n$−leaved rose.

Rotation. A rigid motion in which a rigid body or a figure moves in such a manner, that every point of the body or figure turns about a common centre in a plane or a common line in space. In case of rotation every point turns through the same angle. The rotation is positive if counterclockwise and negative if clockwise.

Rotation of Axes. Syn. *Transformation of Axes.* In two dimensional coordinate geometry if the coordinate axes are rotated through an angle θ (keeping the origin same), then coordinates of a point $P(x, y)$ with reference to the original axes change to (x', y') with reference to the new axes. The relationship between the transformed coordinates is given by

$$x=x'\cos\theta-y'\sin\theta \text{ and } y=x'\sin\theta-y'\cos\theta$$

Rounding off. The method of adjusting the last digit in a number after retaining the specified number of digits. The truncation error becomes less by rounding off the last digit. While rounding off the last digit the following rule is applied. "*Increase the last digit retained by 1 when the first truncated*

figure is 5 or greater then 5, but leave it unaltered when the fi rejected figure is less than 5". For example, the numbe 1.71374 and 27.18917 on truncation of last digit are taken 1.7137 and 27.1892 respectively.

Routh's Rule. The moment of inertia of a homogeneo body about any one of the three perpendicular axes symmetry passing through the centre of mass is given by

(*i*) the product of the mass and one third of the sum the squares of the other two semi-axes, in the case of rectangular body. The moment of inertia of a unifor rectangular lamina of mass m, length l and breadth b about a axis perpendicular to its plane and passing through the cent of the mass is

$$m\left[\frac{(l/2)^2+(b/2)^2}{3}\right]=m\left(\frac{l^2+b^2}{12}\right).$$

(*ii*) the product of the mass and one fourth of the sum the squares of the other two semi-axes, in the case of a circula or an elliptic body. The moment of inertia of a unifor circular disc of mass m and radius r about an axis perpend cular to its plane and passing through its centre is

$$m\left(\frac{r^2+r^2}{4}\right)=\frac{mr^2}{2}.$$

(*iii*) the product of the mass and one fifth of the sum of th squares of other two semi-axes in the case of a sphere o spheriod. The moment of inertia of a solid sphere of mass m and radius r about its diameter is

$$m\left(\frac{r^2+r^2}{5}\right)=\frac{2mr^2}{5}.$$

Row Equivalence Matrix. If a matrix A is reduced to matrix B by the use of elementary row transformations alone B is row equivalent of A.

Row Matrix. A matrix having a single row. For example [0 2 4 6].

Row Transformation. The following operations on a matrix.

(*i*) The interchange of any two rows.

(*ii*) The multiplication of any row by a non-zero scalar.

(*iii*) The addition of a constant multiple of elements of any row to the corresponding elements of other row.

These operations if applied on coloumns of a matrix are called elementary *coloumn transformations.* The two transformations taken together are known as *elementary transformations.*

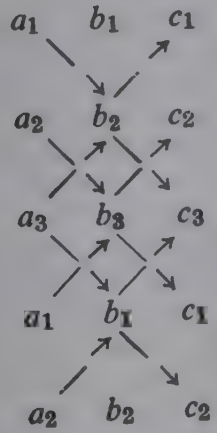
Row Vector. A number of quantities arranged in a single row. For example, the cartesian coordinates of a point (x, y, z). See *Row Matrix.*

Roulette (*i*) The locus of a point which rolls without slipping along a fixed curve, is called *point roulette.*

(*ii*) The locus of the envelope of a straight line attached to a curve which rolls without slipping along a fixed curve, is called a *line roulette.*

The cycloid, epicycloid are examples of roulettes.

Rule of Sarrus. A rule for expanding a determinant. Below the n^2 elements of a nth order determinant, write the



first $(n-1)$ rows as shown in the figure. A descend line along the leading diagonal gives the first positive te the remaining $(n-1)$ descending lines give other positive ter A set of parallel ascending lines give negative terms. Thus, value of the determinant

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = a_1b_2c_3 + a_2b_3c_1 + a_3b_1c_2 - a_3b_2c_1 - a_1b_3c_2 - a_2b_1$$

Runge Kutta Method. A numerical method of solv ordinary differential equations, using iteration procedures.

Saddle Point. A stationary point, which is neither maxima nor a minima of a function $f(x, y)$ of two variabl *i.e.*, it is not a turning point. At the saddle point the first ord partial derivatives $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ both vanish but do n change signs. Thus at this point, the tangent plane is paral to the xy-plane and the surface falls for displacements certain directions and rises for displacements in oth directions.

Sample. A representative finite part of *population*, chose for drawing an inference about certain attributes of th population. Samples may be classified into two categorie (*i*) *Random Sample*. A sample in which every member of th population is equally probable to be chosen. (*ii*) *Stratifie Sample*. A sample which itself consists of random sample taken from each of the several subsets called strata of th population.

Sampling. The process of collecting a sample from the population in statistics for drawing an inference about the population.

Scalar. A physical quantity completely represented by magnitude only without any reference to its direction. The distance, time, speed, weight, temperature are examples of scalars. *Compare Vector.*

Scalar Field. A region which associates a scalar with each point in it.

Scalar Invariant. If $\vec{A}$ is a vector, then divergence $\vec{A}$, denoted by $\nabla \cdot \vec{A}$, is called scalar invariant. *See Vector.*

Scalar Multiplication. The multiplication of a vector by scalar quantity. Thus if $\vec{A}$ is a vector, then $k\vec{A}$ denotes a vector quantity whose magnitude is k times that of $\vec{A}$ and the direction is same as that of $\vec{A}$.

Scalar Product. Syn. *Dot Product.* The scalar product of two vectors $\vec{A}$ and $\vec{B}$ is a scalar quantity, given by

$$\vec{A} \cdot \vec{B} = |\vec{A}|\ |\vec{B}| \cos\theta,$$

where θ is the angle between $\vec{A}$ and $\vec{B}$. The scalar product is commutative, *i.e.*, $\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$. Geometrically the scalar product can be interpreted as magnitude of one vector multiplied by the projection of other vector on the first. The distributive property holds true in this case, *i.e.*,

$$\vec{A} \cdot (\vec{B}+\vec{C}) = \vec{A} \cdot \vec{B} + \vec{A} \cdot \vec{C}.$$

If the vectors $\vec{A}$ and $\vec{B}$ have the components (a_1, a_2, a_3) and (b_1, b_2, b_3) respectively so that

$$\vec{A}=a_1i+a_2j+a_3k \text{ and } \vec{B}=b_1i+b_2j+b_3k,$$

then $\vec{A} \cdot \vec{B}= a_1b_1+a_2b_2+a_3b_3$, where i, j, k are unit vectors along the coordinate axes and $i \cdot i=j \cdot j=k \cdot k=1$ and

$$i \cdot j=j \cdot k=k \cdot i=j \cdot i=k \cdot j=i \cdot k=0.$$

Scale. The markings on the axes of a graph. To plot the given data on the graph paper we choose some distances on the axes of the graph to represent some actual measurement of data. Such an arrangement is called scale. *See Representative Fraction* also.

Scalene Triangle. A triangle whose no two sides are equal.

Scatter Diagram. A diagram representing data from a bivariate frequency distribution (x_r, y_r). For example, the height and weight of n individuals represented as (x_1, y_1), (x_2, y_2), ..., (x_n, y_n), plotted on a graph paper, result in the scatter diagram. A scatter diagram represents the dispersion of a set of data as well as a relationship between the two variables namely x and y. If the points on the graph are evenly distributed about a straight line they are said to be linearly correlated. If the points are distributed evenly around a curve, they are non-linearly correlated ; otherwise uncorrelated. *See Regression Line.*

Schwarz-Christoffel Transformation. A transformation which maps the interior of a polygon of the w-plane into the upper half of the z-plane (xy-plane) and the boundary of the polygon into the real axis. The transformation is

$$\frac{dw}{dz}=A(z-x_1)^{\frac{\alpha_1}{\pi}-1}(z-x_2)^{\frac{\alpha_2}{\pi}-1}\ldots(z-x_n)^{\frac{\alpha_n}{\pi}-1}$$

where $\alpha_1, \alpha_2, \ldots, \alpha_n$ are the interior angles of the polygon having vertices $w_1, \ldots, w_n$, which map into the points $x_1, x_2, \ldots, x_n$ on the real axis of the z-plane.

Schwarz's-Inequality. The inequality

$$\left| \sum_{k=1}^{n} a_k b_k \right|^2 \leqslant \left(\sum_{k=1}^{n} | a_k |^2 \right) \left(\sum_{k=1}^{n} | b_k |^2 \right),$$

where $a_k, b_k \ (k = 1, 2, \ldots, n)$ are any complex numbers.

Secant. (*i*) A straight line that intersects a curve at one or more points, the number depends upon the nature of the curve. See *Chord* also.

(*ii*) See *Trigonometric Functions.*

Secant Hyperbolic. Syn. *Sech.* See *Hyperbolic Functions.*

Second. (*i*) Unit of time (*ii*) A plane angle equal to *1/360th* part of a degree.

Second Order Determinant. See *Determinant.*

Second Order Differential Coefficient. Syn. Second Order Derivative. See *Derivative.*

Second Order Differential Equation. See *Differential Equation.*

Section. See *Cross-Section.*

Sector. (*i*) Part of a circle bounded by two radii and the arc. If r is the radius of the circle and θ the angle in radians between the two radii, then the area of the sector is $\frac{1}{2} r^2 \theta$.

(*ii*) The solid generated by the revolution of a sector of a circle about a diameter of the circle.

Segment. (*i*) Part of straight line or a curve between two of its points.

(*ii*) Part of a plane figure cut off by a straight line

(*iii*) Part of a solid cut off by a plane.

Semi Circle. The part of a circle bounded by a diameter and half the circumference of the circle. The angle in a semicircle is $90°$ or $\frac{\pi}{2}$ radians.

Semi Cubical Parbola. A curve represented by the equations $y^2=ax^3$ or $x^2=ay^3$.

Semi Latus Rectum. Half the length of the latus rectum of a conic section. See *Conic*, *Parabola*, *Ellipse*, and *Hyperbola*.

Separation of Variables. A method used for solving differential equations. For example, if the differential equation $\frac{dy}{dx}=f(x, y)$ can be written as $\frac{dy}{dx}=\frac{f_1(x)}{f_2(y)}$, then the given equation can be solved as $\int f_2(y)\,dy=\int f_1(x)\,dx+c$.

See *Differential Equation.*

Sequence. A set of elements written according to some definite rule, in such a way that each element can be classified as a function of its position. For example in the sequence $\{3, 6, 9, 12, \ldots\}$, the n^{th} term can be written as $3n$, n being a positive integer. Giving different values to n, different terms of the sequence can be obtained. The sequence in which, from and after some fixed term, all the terms are alternately positive and negative, is called *Alternating Sequence.* A sequence $\{u_n\}$, such that $|u_n| \leqslant k$, where k is a finite number, is called a *Bounded Sequence.* A sequence $\{u_n\}$, such that $u_n \leqslant u_{n+1}$ (for all n) is called *Montonic Increasing Sequence* and if $u_n \geqslant u_{n+1}$ (for all n) the sequence is called *Montonic Decreasing Sequence.* However, if $u_n < u_{n+1}$, the sequence $\{u_n\}$ is *Strictly Increasing* and if $u_n > u_{n+1}$, the sequence is *Strictly Decreasing.*

Series. The algebraic sum of the terms of a sequence. Each term of the series can be written as a function of its

position in the series. A series $u_1+u_2+u_3+\ldots+u_n\ldots$, *i.e.*, (Σu_n) is said to be *Convergent* if the sum of its first n terms tends to a finite limit as $n\to\infty$ and *divergent* if the sum of its first n terms approaches infinity, as $n\to\infty$. See *Convergence, Divergence, Sequence.*

Set. A well defined collection of *objects.* These objects are called *members* or *elements* of the set. A set is usually denoted by a capital letter, say, A, B, C, ... A set may be represented by *Tabular or Roster method*, in which all the elements are listed within *braces i.e.* { }, each member being separated by a comma, for example, $A=\{1, 2, 3, 4\}$. A set can also be represented by *Selector or Rule method.* In this method, a set is represented by describing its elements in terms of its characteristic property or properties written within { }. The set of all natural numbers may be written as $\{x : x$ is a natural number$\}$, which reads as, "the set of all x such that x is a natural number". The symbol : denotes "*such that*".

Set Theory. If an element x belongs to a set A, we write $x\in A$. A set containing all the objects likely to be considered in a particular discussion, is called a *Universal set* and is usually denoted by U. If a set A is fully included in a set B, then A is a *Subset* of B and this fact is denoted by $A\subseteq B$ or $B\supseteq A$. If every element of a set A is also a member of another set B, but every element of set B is not a member of the set A, then A is called a *Proper subset* of B and denoted by $A\subset B$ or $B\supset A$. A set having no elements is called a *Null* or *Empty* set. *Complement* of a set A with reference to a universal set U, is a set of all those elements of U, that do not belong to A, symbolically written as A^c or A' or $\sim A$. A set is *finite* or *infinite* according as it contains finite or infinite number of elements. *Equal sets*. Two sets having same distinct elements. For example, $A=\{a, e, i, o, u\}$ and $B=\{a, e, i, o, u\}$. *Equivalent sets*. Two sets having the same number of distinct elements. For example, $A=\{x, y, z\}$, $B=\{1, 2, 3\}$. *Equal sets are always equivalent but converse is not true. Union of two sets A and B* : The set of those elements which belong either to A or to B or to both, denoted by $A\cup B$. *Intersection of sets A and B* : The set of those elements which belong to both A and B, denoted by $A\cap B$.

Overlapping Sets, are those which have at least one commo element and *disjoint sets* are those which do not have an common element. The operations Union and Intersection o sets are *associative*, *commutative* and *distributive*. See *Algebra o Sets*.

Set of Sets. A set whose elements themselves are sets For example $\{\{1\}, \{2, 3\}, \phi\}$.

Sexagesimal System. A system of measurements o angles, in degrees, minutes and seconds. The base of the syste is 60.

1 degree = 60 minutes

1 minute = 60 seconds

The notation for minute (′) and for second (″). See *Angle*.

Shares. See *Stocks and Shares*.

Signed Minor. Syn. *Cofactor in a Determinant*.

Significance Test. In statistics, the criteria for decidin whether a sample taken from a population deviates significantl from the normal or not.

Significant Figures. The specified number of digi which are accepted to denote the approximate value of th number, expressed as an integer and/or a decimal numbe For example, a distance of 784325 km. can be taken a 784300 km. which is correct to four significant digits, the la two zeros are not significant as they denote the order o magnitude only. Similarly, the number of significant digits i ·000785 is 3, as the zeros in the begining are used to deno the place value of the digits. *Thus zeros at the end of a who number or decimal may or may not be significant according the degree of accuracy required but zero on the left is never th first significant figure.*

Similar Figures. Geometrical figures which have sam shapes but different sizes. More precisely, two or more figur are said to be similar if a mapping can be defined between the such that the ratio of distances between pairs of points of on

figure remains same for corresponding points of the other figure.

Similar Fractions. Fractions having the same denominator. For example, $\frac{2}{5}$, $\frac{3}{5}$ or .009 and .023, are similar.

Similar Terms. Syn. *Like Terms.* Terms in an expression which differ in their coefficients only. Similar terms can always be combined into a single term. For example, $7x^2yz+3x^2yz$, are like terms and hence combine to give a single term $10x^2yz$.

Similar Triangles. Two triangles are similar, if either of the following conditions are satisfied, (*i*) the angles of one triangles are equal to the corresponding angles of the other. (*ii*) Three sides of one triangle are proportional to the corresponding sides of the other (*iii*) One angle of one triangle is equal to the corresponding angle of the other and the sides forming the angle of the first triangle are proportional to the corresponding sides of the other. The above three conditions for similarity of triangles are popularly known as (*i*) *A A A* postulate (*ii*) *SSS* postulates (*iii*) *SAS* postulate, respectively. Compare *Congurence.*

Simple Harmonic Motion (S.H.M.). A motion in a straight line in which the accceleration is proportional to the distance from some fixed point in the line and is always directed towards the fixed point. A simple pendulum, a fork, a spring all are examples of S.H.M. A body which is to produce simple harmonic motion when moved away from its central position expriences a restoring force proportional to the displacement from zero position. The equation of motion may be written in the form. $m\frac{d^2x}{dt^2}=-\lambda x$ where λ is the constant of proportionality and m the mass of the body. During the motion there is an exchange of potential and kinetic energy, their sum remaining constant under the assumption of absence of damping force. It ω is the angular velocity, the period T is given by

$$T=1/f=2\pi/\omega,$$

where f is the frequency. The equation of motion is

$$x = x_0 \sin \omega t,$$

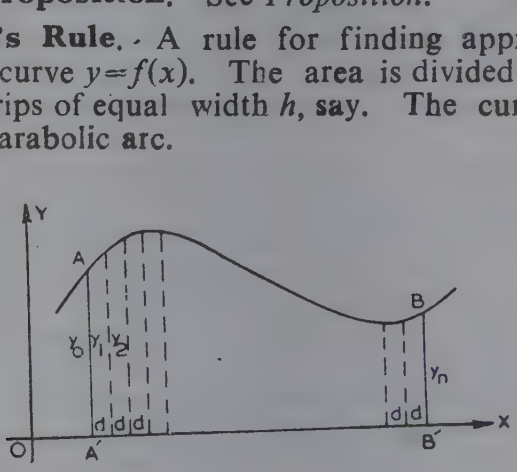
which is a sine wave, x_0 being the maximum displacement.

Simple Interest. See *Interest.*

Simple Pendulum. See *Pendulum.*

Simple Proposition. See *Proposition.*

Simpson's Rule. A rule for finding approximately t area under a curve $y=f(x)$. The area is divided into an ev number of strips of equal width h, say. The curve is appro mated by a parabolic arc.



The value of the integral

$$\int_a^b y\,dx = \int_a^b f(x)dx$$

is the area *ACOBA* which is approximately equal to

$$\frac{h}{3}\left[y_0+y_n+4\ (y_1+y_3+\cdots\cdots)+2(y_2+y_4+\cdots\cdots)\right.$$

This gives a better approximation to the actual value tha that by trapezoidal rule.

Simultaneous Equations. Two or more equations th are true at one and the same time and are therefore satisfied b the same values of the unknowns involved. For example, th equations.

$$2x+y=3$$
$$3x+4y=2$$

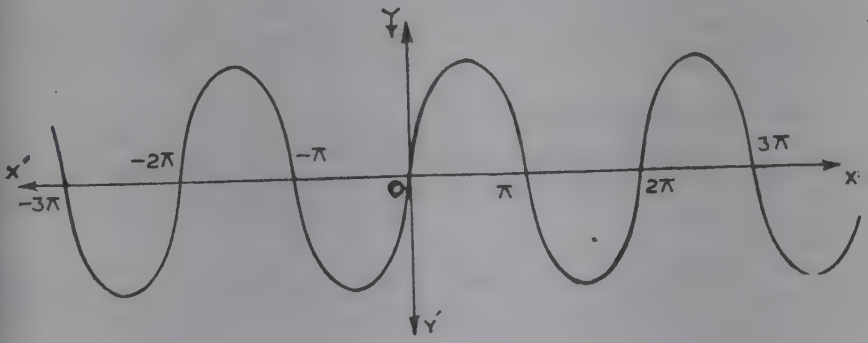
are simultaneous equations having solution $x=2$, $y=-1$.

Sin. Abbreviation for *Sine.*

Sinh. Abbreviation for *Sine Hyperbolic.*

Sine. See *Trigono-metric Ratios.*

Sine Curve. The curve given by the equation $y=\sin x$. It is a periodic curve with period 2π. Here $-1 \leqslant y \leqslant 1$.



The curve is also known as *sine wave.*

Sine Formula or Sine Rule. In any triangle the ratio of the length of a side to the sine of the opposite angle always remains constant. If in a triangle ABC, $\angle A$, $\angle B$, $\angle C$ are angles opposite to the sides of lengths a, b, c, respectivelv then $a/\sin A = b/\sin B = c/\sin c$.

Sine Hyperbolic (sin h). See *Hyperbolic.*

Single-Valued Function. A function $y=f(x)$ is called single-valued if for one value of x, y has only one value.

Singular Matrix. A square matrix whose corresponding determinant has the value zero. A singular matrix does not have inverse.

Singular Point. A point at which a function of a complex variable ceases to be analytic. See *Analytic Function.*

Sinusoidal. A quantity whose graph is shaped like a sine curve.

Skew Lines. In space there can be drawn lines which neither parallel nor intersecting. The distance between any t of them is actually the shortest distance between them which perpendicular to both the lines.

Skewness. A measure of the degree of symmetery o distribution. If the frequency curve for a given distribution long tail to the right (left) and a short tail to the left (right) i said to possess positive (negative) skewness. The skewness measured by the difference of mean and mode divided by t standard deviation.

Slant Height of Cone. The length of a generating li (element) of a right circular cone, that is, the distance of a point on the circumference of the base from the vertex of t cone. See *Cone.*

Slant Height of Pyramid. The altitude of the late faces of a right pyramid. See *Pyramid.*

Slide Rule. A calculating device consisting of two loga thmic scales which can slide relative to each other. These a used for multiplication and division of two numbers by t addition and substraction of the logarithms of the numbe which are represented as lengths on the scales.

Slope. See *Gradient.*

Small Circle. The section of the surface of a sphere a plane which does not pass through the centre of the sphe Compare *Great Circle.*

Smooth Surface. A surface for which the reaction betwe it and a body moving on it in any way is always along t normal to the surface, the friction between the surface and t body is to be neglected in the calculations.

Solid Angle. The analogue of a plane angle in thr dimension, the angle subtended at a point by a given surfa (unlike by a line). Its unit is *steradian* which is define analogously to the radian, as the solid angle, substendi unit area at unit distance. The solid angle corresponding to o revolution (2π radians) is 4 steradians.

Solid Geometry. The study of geometrical conifigurations in space.

Solid of Revolution. Solid generated by the revolution of a plane area about a line called *axis of revolution.* The perimeter of the arc on revolution generates a *surface of revolution.* By the application of integral calculus the volume of the solid of revolution and the area of the surface of revolution can be calculated.

Solution of Equation. See *Root of Equation.*

Solution of Quadratic Equation. See *Quadratic Equation.*

Solution of Simultaneous Equations. See *Cramer's Rule.*

Spearman's Co-efficient of Rank Correlation. Measurement of degree of association between the ranks of certain objects using two attributes x and y giving the data $(x_1, y_1), \cdots, (x_n, y_n)$. The objects are ranked with respect to the two attributes first with respect to x and then to y. d_i's denote the difference the ranks for each object. The Spearman's coefficient of rank correlation f is given by

$$f=1-\frac{6\Sigma d_i^2}{n(n^2-1)}.$$ See *Rank.*

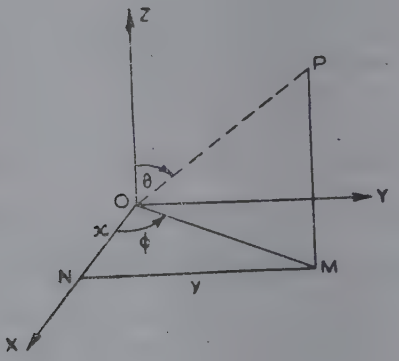
Specific Gravity. Syn. *Relative Density.*

Speed. Rate of change of displacement with respect to time without considering the direction. Speed with direction becomes velocity. See *Velocity.*

Sphere. A solid generated by the revolution of a semicircular area about its bounding diameter. Its volume is $4/3\ \pi r^3$ where r is its radius. If the centre of the sphere is taken as the origin of cartesian coordinate system then the surface of the sphere will have the equation. $x^2+y^2+z^2=r^2$.

Spherical Cap, Zone. The portion of the surface of a sphere lying between two parallel plane sections is called *spherical zone.* A spherical zone in which one plane is tangential to the spherical surface, is called spherical Cap.

Spherical Polar Coordinates. The polar coordinates which define the position of a point P in space are given in terms of radial distance r from a fixed point O called origin



the angle ϕ which is the angle made by the projection of OP on the $XOY-$plane with the axis of x, and the angle θ made by OP with the z-axis, that is (r, θ, ϕ). Let P have the cartesian coordinates (x, y, z) then

$$x=r\cos\theta\cos\phi, \quad y=r\cos\theta\sin\phi, \quad z=r\sin\theta$$

Spherical Sector. See *Sector.*

Spherical Segment. The set of all the points equally distant from one fixed point called centre. The common distance is called radius of the surface. Actually, this spherical surface can be obtained by revolving the arc of semi-circle about its bounding diameter. The area of the surface with radius r is equal to $4\pi r^2$ where r is the radius of the semi-circle. See *Sphere.*

Spherical Triangle. A portion of a spherical surface bounded by the arc of three great circles. The sum of its spherical angles is 180 to 540. See *Great Circle.*

Spherical Trigonometry. The study of properties of spherical triangles and their measurements.

Spheroid. Syn. *Ellipsoid.*

Spiral. A curve described by the rotation of a point about a fixed point in such a fashion that its distance r, the radius vector, from the fixed point, the pole, is related to the angle of rotation θ, the vectorial angle. For example, the spiral of Archimedes $r=a\theta$, equiangular (logarithmic) spiral $r=ae^{\theta \cot \alpha}$

Square. (1) The second power, denoted by the exponent 2, as x^2 standing for $x \cdot x$. (2) A plane figure in geometry in the form of a quadrilateral with equal sides and four right angles.

Square Root. Square root of a number is that number which when multiplied by itself produces the given number. It can be taken as the solution of the equation $x^2-a=0$ where a is the number given. If a is positive and real, x will have two real values with opposite signs and equal magnitude. If a is negative, x will have two imaginary values of equal magnitude but opposite signs.

Square matrix. A matrix having equal number of rows and columns. The diagonal from top left to bottom right is called the *leading* or *principal diagonal.* The sum of the elements in the leading diagonal is called *trace* of the matrix. See *Matrix.*

Squaring the Circle. To construct a square which has the same area as that of a particular circle. An exact solution is not possible.

Stable, Unstable Equilibrium. See *Equilibrium.*

Standard Deviation. (σ). The square root of the mean of the squares of deviations from their average, used as a measure of dispresion in Statistics. It is given by

$$\sigma=\sqrt{\frac{1}{n}\Sigma(x_i-\bar{x})^2}$$

Standard Form of Number. A number written as a number between 1 to 10. For instance, the numbers 2145.71 and 0.073534 can be written in standard form as 2.14571×10^3 and 7.3554×10^{-2}.

Statics. A branch of Mathematics dealing with the equ brium of a body under the action of forces. In the state equilibrium the resultant force and torque are both zero.

Stationary Point. A point on a curve at which tangent will have zero slope and hence parallel to x-axis. the turning points (maxima and minima) are station points.

At a stationary point the derivative obtained from equation $y=f(x)$, will be zero. If the second derivative the stationary point is negative then the maxima exists.

Statistical Inference. See *Sampling.*

Statistics. The study of the methods of collecting a analysing data. It is itself a vast subject with various aspe of study as classification of data, inference involving probabil and probability distributions, sampling, etc.

Steradian. The unit of measuring a *solid angle.*

Stirlings Theorem. For very large values of n

$$n\,!=n^n\,e^{-n}\sqrt{2\,\pi n}$$

aproximately. See *Factorial.*

Stochastic Process. A process which produces a ser of random values of a variable and hence a statistical distrib tion from them.

Stocks and Shares. The money borrowed by the gover ment or a trading company is called the stock. It is issued the forms of bonds, promissory notes, securities, conso debentures, etc. The shares of trading companies are anoth form of stocks. Persons holding these stocks are call share holders. Profit divided among the share holders is call *dividend.*

Straight Line. The line which passes through two poin in such a way that the line segment between the points minimum. In general, the term line means straight line.

Strain. A measure of deformation in an elastic body which results in stresses. See *Bulk modulus*, *Young's Modulus*.

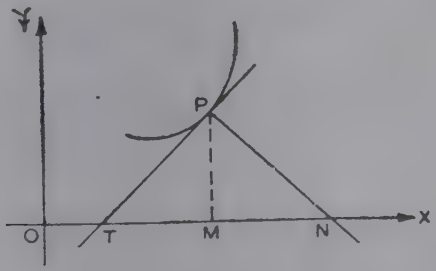
Stratified Sample. See *Sample*.

Stress. On the application of external force on an elastic body deformation takes place and because of that stresses develop. Stress can be conceived as internal forces per unit area in a body which suffered strains. Stresses cannot be there in rigid bodies and can be witnessed in elastic, plastic bodies. Stresses may be *tensile* or *shear*.

Subfield. A subset of a field which it self is a field. See *Field*.

Subgroup. A subset of a group which itself is a group. See *Group*.

Subnormal, Subtangent. From any point P on a curve we draw tangent and normal to meet the x-axis at T and N. in the figure, respectively. MN & TM are subnormal and subtangent for the curve at the point P. PT and PN are called the lengths of tangent and normal at P. TM and MN can be regarded as orthogonal projections of PT and PN on the axis of x respectively.



Subroutine. A section of a programme which requires repetitive use in different parts of the main programme for solving a problem by the computer. In the main programme, as soon as a subroutine is encountered the control is shifted to the subroutine with specified values of the input parameters. The

subroutine has output parameters or arguments also in which the results of computations during the execution of the subroutine are stored. Usually, a program consists of many subroutines which are extremely useful. See *Programe*.

Subset. See *Set Theory*.

Subtangent. See *Subnormal, Subtangent*.

Subtend. If the points A and B of a chord AB of a curve are joined to another point P on the curve, then the arc AB or chord AB is said to subtend an angle $\angle APB$ at the point P. The diameter of a semi-circle always subtends a right angle at every point on the circumference. An arc or chord of a circle subtends an angle at the centre twice that at the circumference.

Subtraction. (Symbol $-$). A binary operation of finding the remainder when a number is removed from another. It is a reverse process of addition and can be described as finding the difference between two quantites. It is of the form $x-y$. It can be applied to matrices, vectors, complex numbers as well. See *Vector, Matrix, Complex numbers*.

Successive Differentiation. Repeated differntiation to higher order derivatives. For example, if

$$\frac{dy}{dx}=f'(x), \text{ then } \frac{d^2y}{dx^2}=f''(x) \text{ etc.}$$

In general, nth order derivative is denoted by $\frac{d^ny}{dx^n}$ or $y^{(n)}$. See *Differentiation*.

Sum. Result of adding two or more quantities.

Sum of Vectors. See *Vector*.

Sum of Sets. Syn. *Union of Sets*. See *Set Theory*.

Sum to Infinity. Summation of infinite number of terms. It is possible for an infinite series. See *Convergent Series*.

Summation Sign. The Greek capital letter Σ is used to denote the summation of terms of a series. The sum of the first n terms of a sequence $\{u_r\}$ is given by

$$\sum_{r=1}^{n} u_r = u_1 + u_2 + \cdots + u_n,$$

whereas $\sum_{n=1}^{\infty} u_n$ stands for the sum of infinite number of terms of an infinite series.

Superficial. On the surface only.

Supplementary Angles. A pair of angles whose sum is a straight angle or two right angles, or 180° or π radians. One angle is known as the supplement of the other. Compare *Complementary Angles.*

Surd. A numerical expression containing an irrational number. Surds are quadratic, cubic, quartic, etc., depending upon the power involved, *i.e.*, 1/2, 1/3, 1/4 etc. For example, $\sqrt{3}$, $\sqrt{7}$.

These are also known as *pure surds* while a *mixed surd* involves a rational number also. For example $2+\sqrt{3}$ or $5\sqrt{\pi}$.

Surface. A set of points which form a space having two dimensions only. Surface may be plane like that of circular plate or curved like that of a sphere.

Surface of Revolution. See *Solid of Revolution.*

Symmetric Difference. The symmetric difference of two sets A and B is the set of elements which belong either to A or to B, but not to both.

Symmetric Equation. An equation consisting of only symmetric functions or expressions. See *Symmetric Function.*

Symmetric Function. A function in two or more variables which remains unchanged by interchanging any two variables. For example, the function $x^3+y^3+z^3-3x^2y^2z^2$.

Symmetric Functions of Roots. If $\alpha_1, \alpha_2, \ldots \alpha_n$ are the roots of the equation $a_0x^n+a_1x^{n-1}+a_2x^{n-2}+\ldots+a_n=0$, then the sum of the roots taken one at a time is $-a_1/a_0$, taken two at a time is a_2/a_0 ; taken three at a time is $-a_3/a_0$, and so on. Thus, for a quadratic ; $\alpha_1+\alpha_2=-a_1/a_0$, $\alpha_1\alpha_2=a_2/a_0$, and for a cubic $\alpha_1+\alpha_2+\alpha_3=-a_1/a_0$, $\alpha_1\alpha_2+\alpha_2\alpha_3+\alpha_3\alpha_1=a_2/a_0$, $\alpha_1\,\alpha_2\,\alpha_3 = -a_3/a_0$.

These are all known as symmetric functions of the roots.

Symmetric Group. See *Group*.

Symmetric Relation. A relation R defined on a set S is called symmetric if for any two elements a and b of S, $a\ R\ b$ implies $b\ R\ a$.

Symmetry. Two points A and B are symmetrical or are said to possess symmetry about a plane, a line or a point, if A is reflection of B and also, B is a reflection of A about the plane, the line or the point.

System Analysis. The thorough study of the working of an organisation or a system, the purpose for which it is meant and its basic requirements in order that a system analyst may, improve upon the performance of the system. The system analyst is expected to reduce the total number of steps involved.

T

Tan. Abbreviation of tangent. See *Trigonometric Functions.*

Tan h. Abbreviation of tangent hyperbolic. See *Hyperbolic Function.*

Tangent (1) See *Trigonometric Functions* (2) A straight line or a plane that touches a curve or a surface respectively without cutting it. In plane geometry the slope of the tangent to the curve is dy/dx and is also equal to $\tan \psi$ where ψ is the angle made by the tangent line with the positive direction of the x-axis. See *Tangeet Line, Tangent Plane.*

Tangent Hyperbolic. See *Hyperbolic Function.*

Tangent line. Tangent line at a given point P on a curve is the limiting position of a secant PQ when the point Q approaches the point P along the curve.

Tangent Plane. A plane touching a surface at a point P, called the point of contact, so that any line through P in this plane is a tangent line to the surface at the point P.

Taylor's Series (Theorem). If a function $f(x)$ of one variable x is capable of being expanded about a point $x=a$, in a powers of $(x-a)$ and is continuously differentiable any number of times at $x=a$, then the expansion of $f(x)$, about the point $x=a$, is given by

$$f(x)=f(a)+(x-a)f'(a)+\frac{1}{2!}(x-a)^2 f''(a)+\frac{1}{3!}(x-a)^3 f'''(a)+\cdots$$

It is in the form of an infinite power series.

A particular case of it when $a=0$, is called *Maclaurin's series* or *expansion* given by

$$f(x)=f(0)+x\,f'(0)+\frac{x^2}{2!}\cdot\,''(0)+\frac{x^3}{3!}f'''(0)+\cdots$$

Tension. Internal force of a body which results in its stretch, that is resisting the external forces applied to the body to increase its length.

Tensor. A generalization of a vector of two or three dimensions to n-dimensional entity. Tensors depict how all the components of a vector quantity behave while undergoing certain transformations. See also *Vectors.*

Terminal. A device by which a computer user can ha a direct access to the computer even while sitting at a distanc By terminal the user feeds the input in the form of data an instruction and gets the output also in the form of results. Th input is fed in the computer through a keyboard similar to th of a typewriter, displayed on the screen, output also appea on the screen. One can make changes in the instructions an data and correct the syntax errors etc.

Terminating Continued Fraction. See *Continue Fraction.*

Terminating Decimal. See *Decimal.*

Tetrahedron. A polyhedron having four faces in th form of triangles.

Theorem. A general conclusion based on certain assump tions which are to be proved or a logical conclusion, proof o which is already known.

Theory of Equations. The study of equations, method of solving the equations, existence or non-existence of root relations between the roots and coefficients.

Theory of Games. Theory of probabilities of varying out comes in games. These are calculated from the rules of th game and the number of players, using techniques of statistics.

Theory of Sets. See *Set Theory.*

Theory of Indices. It spells out the laws for multiplicatio and division of quantities which are powers of the sam quantity, expressed as follows,

$$a^p\ a^q = a^{p+q}, \quad a^p/a^q = a^{p-q}, \quad (a^p)^q = a^{pq}.$$
$$\sqrt[n]{a^m} = a^{m/n}, \quad a^1 = a, \quad a^0 = 1, \quad a^{-m} = 1/a^m.$$

Three Dimensional Geometry. See *Solid Geometry.*

Time Sharing. A technique in computers which facili tates a number of jobs to be run simultaneously instead o

one after another. What actually happens is that each program is transferred in turn from the backing store to the main store and allowed to run for a short time, doing a specified job. This is particularly useful for the programs controlled by users sitting at the terminals. Thus, quite a good number of users can interact with the computer almost simultaneously.

Ton. British unit of weight equal to 2240 pound, *i.e.*, equivalent to 1016 kg.

Tonne. (metric) A metric unit of weight equal to 1000 Kg.

Topological Space. Let T be a set of all subsets of a set S (including X and ϕ the null set X), will be called a topological space if it satisfies the condition

$$A \in T,\ B \in T, \text{ then } (A \cup B) \in T, \text{ and } (A \cap B) \in T.$$

The elements of T are known as open sets of the topological space X. The topology of a topological space is defined by the properties of its open sets. See *Topology*.

Topology. A branch of modern mathematics rather of geometry, concerned with the general properties of shapes and space. In topology we study the properties that are not altered by continuous distortions such as stretching or twisting. For example, a sphere and an ellipsoid are though different figures in geometry but are equivalent figures in topology since one can be transformed into the other by a continuous deformation.

Torque. The turning effect of a force acting on a body, and is measured by the moment of the force F about an axis of rotation as $F \times s$, where s is the distance from the axis to the line of action of the force.

Torus (Anchor Ring). The solid generated by the revolution of a circular area about an axis in its plane but not intersecting it. It is like the inner tube of a tyre.

Total Derivative. If F is a function of two variables x and y where x and y are separately some functions of a third variable t then the total derivative of F with respect to t is

given, in terms of partial derivatives of F with respect to x and y as

$$\frac{dF}{dt} = \frac{\partial F}{\partial x}\frac{dx}{dt} + \frac{\partial F}{\partial y}\frac{dy}{dt}$$

This can be easily extended to the case in which F is a function of more than two variables.

Total Differential. If F is a function of two independent variables x and y, and x and y are given infinitesimal changes δx and δy respectively, then the infinitesimal change δF in F is given by

$$\delta F = \frac{\partial F}{\partial x}\,\delta x + \frac{\partial F}{\partial y}\,\delta y.$$

Trace. See *Square Matrix.*

Tracing. See *Curve Tracing.*

Trajectory. Path traversed by a projectile. The trajectories of heavenly bodies are called *orbits.*

Transcendental Functions. Functions which are not algebraic. The circular, exponential, and logarithmic functions are examples of transcendental functions.

Transcendental Numbers. Numbers which are not algebraic. For example, π, e, $(3)^{\sqrt{2}}$

Transformation. See *Correspondence, Mapping, Function*

Transformation of Axes. It can be done in two ways. Axes of Coordinates are said to be *translated* when the origin of coordinates is shifted to a new position, axes remaining parallel. Axes are said to be *rotated* when these are turned through the same angle. If the origin is shifted to a point (h, k) the original coordinates of a point $P(x, y)$ become (X, Y) with respect to new axes, such that $x = X + h$, $y = Y + k$. See *Rotation of Axes.*

Transitive Relation. A relation R defined in a set such that for any three elements a, b, and c belonging to the set aRb, bRc implies that aRc.

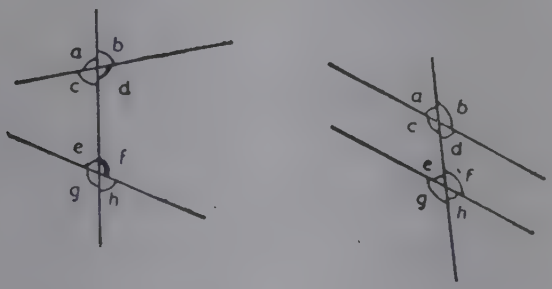
Transitivity. See *Order Relation, Transitive Relation.*

Translation. The change of position of a geometrical figure without any change in its direction.

Translation of Axes. See *Transformation of Axes.*

Transpose of Matrix. The matrix obtained by interchanging the rows and columns of a given matrix. It is usually denoted by A^T or A', where A is the original matrix. Also $(A')'=A$.

Transversal. A straight line which crosses *Coplanar lines.* Eight angles are formed when a transversal cuts two such lines as shown in the figure. These angles are paired and catagorised as follows,



Alternate Angle : (c, f), (d, e).

Corresponding Angles : (a, e), (b, f), (c, g), (d, h).

Vertically Opposite Angles : (a, d), (b, c), (e, h), (f, g).

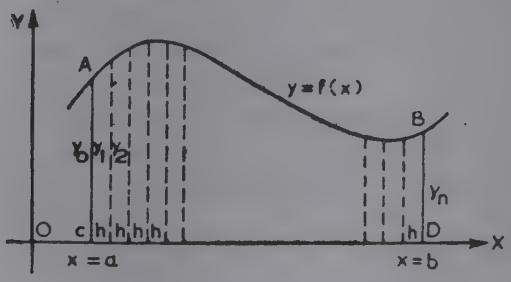
When a transversal cuts two parallel lines, all above mentioned pairs of angles are equal.

Transverse Axes. See *Hyperbola.*

Trapezium. Syn. *Trapezoid.* A Quadrilateral having two sides as parallel.

Trapezoidal Rule. A rule of finding the approximate area bounded by a curve by dividing it into a number of trapeziums forming vertical columns of equal widths with

bases on the axis of x. It gives a numerical method of integration. The area of the portion $A\ B\ B'\ A'\ A$ is equal to $\frac{1}{2}h(y_0+2y_1+2y_2+\ldots+2y_{n-1}+y_n)$ where h is the length of equal sub-intervals and $y_0, y_1, y_2 \ldots\ldots, y_n$ are the $(n+1)$ ordinates. See also *Simpson's Rule*.



Triangle. A plane figure bounded by three sides, or a polygon with three sides. Sum of its interior (or vertical angles) is 180° or π radians. These are of following six types.

Acute Angled. All the angles are less than 90°.

Obtuse Angled. One angle is greater than 90°.

Right Angled. One angle is of 90°.

Isosceles. Two sides are equal, hence two angles opposite the equal sides, are equal.

Equilateral. All sides are equal, hence all angles are 60°.

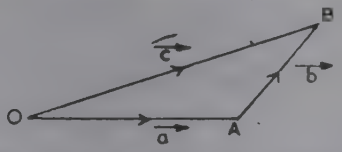
Scalene. No two sides are equal.

Triangle Inequality. The property of any plane triangle that any side is always less than the sum of the other two sides.

Triangle of Forces (Velocities). See *Triangle of Vectors*.

Triangle of Vectors. A diagramatic representation of *composition* of vectors. If vectors $\vec{a}$ and $\vec{b}$ acting at a point O are represented in magnitude and direction by the line segment

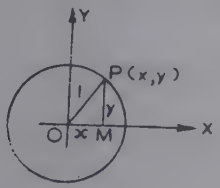
$\overrightarrow{OA}$ and $\overrightarrow{AB}$ then their resultant vector $\overrightarrow{c}$ is represented by $\overrightarrow{OB}$ along the third side of the triangle OAB. In the diagram $\overrightarrow{c}$ is the vector sum of $\overrightarrow{a}$ and $\overrightarrow{b}$ written as $\overrightarrow{c}=\overrightarrow{a}+\overrightarrow{b}$, $\overrightarrow{a}$ is the vector difference of $\overrightarrow{c}$ and $\overrightarrow{b}$. $(\overrightarrow{a}=\overrightarrow{c}-\overrightarrow{b})$ and $\overrightarrow{b}$ is the vector difference of $\overrightarrow{c}$ and $\overrightarrow{a}$ $(\overrightarrow{b}=\overrightarrow{c}-\overrightarrow{a})$. Consequently, if three vectors acting at a point can be represented by the line segments $\overrightarrow{OA}$, $\overrightarrow{AB}$, $\overrightarrow{BO}$ along the sides of a triangle OAB, then their resultant will be zero and the triangle system will be in equilibirium.



Triangular Matrix. A square matrix in which all the elements either above or below the leading diagonal are zero. If all the elements above the leading diagonal are zero the matrix is called lower triangular and if all the elements below the leading diagonal are zero, the matrix is called *upper triangular.* The value of the determinant of a triangular matrix is equal to the product of its leading diagonal elements.

Trichotomy. See *Order Relation.*

Trigonometric Functions. Syn. *Circular Functions.* Functions of angles defined in terms of trigonometric ratios. If a point $P(x, y)$ lies on the circumference of a unit circle



$x^2+y^2=1$; with centre O, the origin, the trigonometric functions of the angle θ made by OP with positive direction of x-axis, are given by

$$\sin\theta = y,\ \cos\theta = x,\ \tan\theta = \frac{y}{x},\ \cot\theta = \frac{x}{y},\ \sec\theta = \frac{1}{x},$$

$$\operatorname{cosec}\theta = \frac{1}{y}.$$

See *Reduction Formula in Trigonometry.*

Trigonometric Identities. Identities involving trigonometric functions. For example,

$\sin^2\theta + \cos^2\theta = 1$, $\sec^2\theta = 1 + \tan^2\theta$, $\operatorname{cosec}^2\theta = 1 + \cot^2\theta$ etc.

Trigonometric Series. A series involving trigonometric functions, expressed as

$$\tfrac{1}{2}a_0 + (a_1\cos x + b_1\sin x) + (a_2\cos 2x + b_2\sin 2x) + \cdots$$

Fourier Series is the most important trigonometric series.

Trigonometry. Measurement and study of properties of triangles and their angles.

Trinomial Expression. See *Multinominal.*

Triple Integral. An integral of the form

$$\iiint f(x, y, z)\, dx\, dy\, dz,$$

carried out with respect to three variables over different regions is called triple integral. See *Multiple Integral.*

Triple Point. See *Multiple Point.*

Triple Product. Product of three vectors. See *Triple Scalar Product, Triple Vector Product.*

Triple Root. See *Multiple Root, Repeated Root.*

Triple Scalar Product. A product of three vectors, the result of which is a scalar quantity. For vectors $\vec{a}, \vec{b}, \vec{c}$ it is defined as $\vec{a}.(\vec{b}\times\vec{c})$. It is denoted by $[\vec{a}\ \vec{b}\ \vec{c}]$ and is also called *box* product Geometrically it is equal to the volume of parallelopiped with $\vec{a}, \vec{b}, \vec{c}$, forming its coterminus edges. The box product is zero if the three vectors are coplanar.

Triple Vector Product. The product of three vectors the result of which is vector quantity. It is defined, for three vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$, as

$$\vec{a}\times(\vec{b}\times\vec{c})=(\vec{a}\,.\,\vec{c})\,\vec{b}-(\vec{a}.\vec{b})\,\vec{c}$$

This can also be written as

$$(\vec{a}\times\vec{b})\times\vec{c}=(\vec{a}\,.\,\vec{c})\,\vec{b}-(\vec{b}\,.\,\vec{c})\,\vec{a}$$

Trisection. Dividing into three equal parts.

Trivial. A solution to one or a set of equations which is obvious. For example, the equation $x^3+y^3=3x+7y$ has $x=y=0$, as a trivial solution.

Truncated Solid. A solid obtained by taking its section by two planes.

Turning Point. A point on a curve at which the slope of the tangent line to a continuous curve, changes its sign. All turning points are stationary points and give a maxima or a minima. See *Stationary Points, Maxima. Minima.*

Two Dimensional. A figure in a plane having length, breadth but no height or depth. Circle, Parabola, Ellipse, Hyperbola, Rectangle, Triangle etc., all are examples of two dimensional figures.

U

Unbounded. A function which is either not bounded above or not bounded below or both is known as unbounded function. For example, 3^x is not bounded above but has greatest lower bound as zero. See *Lower Bound, Upper Bound.*

Uniform Acceleration. Acceleration which remains constant throughout the motion of an object.

Uniform Convergence. The functional series

$$u_1(x)+u_2(x)+\ldots\ldots+u_n(x)+\ldots\ldots,$$

is called uniformly convergent on an interval (a, b) if for any small arbitrary $\epsilon>0$, there exists an integer N such that for all $n \geqslant N$, $| s(x)-s_n(x) | <\epsilon$, for any value of x in (a, b), where $s(x)$ and $s_n(x)$ denote the sum of the infinite series and sum to n terms of the series respectively. See *Convergence.*

Uniform Speed. Constant speed.

Uniform Velocity. Velocity which is a vector quantity remains constant, that is, does not change in magnitude as well as in direction.

Uniform Scale. A scale in which equal intervals correspond to equal values of the variables concerned.

Unilateral Surface. A surface which has only one side. See *Mobius Band.*

Union of Sets. See *Set Theory.*

Unit Circle. Circle with radius unity. Its area is π and circumference 2π.

Unit Cube. A cube with each edge of unit length. Its volume is taken as unit of volume.

Unit Fraction. A fraction with numerator as unity. It is of the form $1/n$ where n is any integer.

Unit Matrix. See *Matrix.*

Unit Sphere. A sphere of unit radius. The equation

$$x^2+y^2+z^2=1,$$

represents a sphere of unit radius and centre at the origin.

Unit Vector. A vector unit of magnitude. The unit vectors along the co-ordinate axes are denoted by $\hat{i}$, $\hat{j}$, $\hat{k}$

A unit vector along the direction of the vector $\vec{a}$ is denoted by $\hat{a}$.

Unity. The symbol 1. It represents an abstract number of magnitude 1. In Boolean algebra, theory of groups, rings and fields, it stands as symbol of unit element not necessarily meaning the absolute number 1.

Universal Set. See *Set Theory.*

Unlike Terms. Terms which are not similar (like). These terms can't be reduced to a single term. Compare *Like Terms.*

Unstable Equilibrium. Equilibrium which is such that if it is disturbed a little there is a tendency for the system to move away from its original position rather than to come back to original position of equilibrium.

Upper Bound. See *Bound.*

V

V. Roman Symbol for 5.

Validity. A property of arguments sufficiently supported by facts or reasons. An argument is valid if it is impossible that the conclusion be false.

Value, Absolute. See *Absolute Value.*

Vanish. To equal zero.

Variable. A quantity whose value changes and which is usually denoted by a letter. If a variable can have only

integral values it is called an *integer variable* whereas a variabl wnich can have fractional values also is called a *real variable.* I two variables x and y are such that y can be expressed in term of x and if we allow a change in value of x, the value of y als changes in some fashion, then x is called independent variabl and y dependent variable.

Variance. A measure of dispersion in statistics. It i equal to square of *standard deviation.* See *Standard Deviation.*

Variate. A term used in statistics to denote a variabl quantity.

Variation. Deviation of a variable from its mean. Se *Deviation.*

Vector. It is a quantity which has magnitude as well a the direction which distinguishes it from scalar quantities. Fo example, the displacement, velocity, acceleration, force, etc Vectors are written by putting an arrow overhead of a small o a capital letter *e.g.*, $\vec{a}$ or $\vec{A}$. It is also denoted by antique letter **a** or **A**. Vector algebra is different from algebra of scalar quanti ties. Vectors can also be written in terms of its components In three dimensional coordinate system we denote unit vector along the axes of x,y, and z by i,j, and k respectively. If point p has coordinates $(x.y,z)$ its *position vector OP* is denote by $xi+yj+zk$ where O is the origin. If $\vec{a}$ is a vector, $-\vec{a}$ wi be a vector of magnitude equal to that of $\vec{a}$ but in opposit direction.

Vector Difference. Difference of vectors $\vec{A}$ and $\vec{B}$ de noted by $\vec{A}-\vec{B}$ is equivalent to the sum of $\vec{A}$ and $-\vec{B}$. Se *Composition of Vectors.*

Vector Multiplication. Two vectors can be multiplied i entirely different ways. In one way the result is a scala

quantity hence called scalar product. See *Scalar Product*, *Vector Product*.

Vector Product. Syn. *Cross Product*. The vector product of two vectors $\vec{A}$ and $\vec{B}$ is denoted by $\vec{A} \times \vec{B}$. Its magnitude is $|\vec{A}|\,|\vec{B}| \sin\theta$ where θ is the angle between two vectors and its direction is along the normal to the plane passing through both the vectors and direction sense is determined by the right handed screw system. If the components of $\vec{A}$ and $\vec{B}$ are (a_1,a_2,a_3) and (b_1,b_2,b_3) respectively so that

$$\vec{A}=a_1i+a_2j+a_3k,\quad \vec{B}=b_1i+b_2j+b_3k, \text{ then}$$

$$\vec{A}\times\vec{B}=\begin{vmatrix} i & j & k \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}=i(a_2b_3-a_3b_2)+j(a_3b_1-a_1b_3)+k(a_1b_2-a_2b_1)$$

The product is not commutative, rather $\vec{A}\times\vec{B}=-\vec{B}\times\vec{A}$ Distributive property is valid here so that

$$\vec{C}\times(\vec{A}+\vec{B})=\vec{C}\times\vec{A}+\vec{C}\times\vec{B}$$

The magnitude of $\vec{A}\times\vec{B}$ can be interpreted geometrically as the area of a parallelogram formed by taking vectors $\vec{A}$ and $\vec{B}$ as its adjacent sides.

Vector Projection. It is a vector which is the orthogonal projection of one vector on another. Thus, the vector projection of $\vec{A}$ on $\vec{B}$ is $(a\cos\theta)\,\hat{b}$ where a the is magnitude of $\vec{A}$, $\hat{b}$ is the unit vector in the direction of $\vec{B}$ and θ is the smaller angle between $\vec{A}$ and $\vec{B}$.

Vector, Parallelogram of. See *Parallelogram of Vectors*

Vector, Trianlgle of. See *Triangle of Vectors.*

Vector Space. A set of vectors satisfying the following properties. (1) If $\vec{V}$ is vector and k is a scalar then $k\vec{V}$ belongs to the set, which defines scalar multiplication, (2) There exists an identity vector I so that $\vec{I} \cdot \vec{V} = \vec{V} \cdot \vec{I} = \vec{V}$. (3) Set is commutative with respect to addition, (4) Set obeys the associativity with respect to addition as well as multiplication, (5) Set obeys distributive property of vectors ; such a set of vectors is known as Vector Space. The vector space may be two dimensional, there dimensional, ..., or n dimensional.

Vector Sum. Result of adding two vectors $\vec{A}$ and $\vec{B}$. If components of $\vec{A}$ and $\vec{B}$ are (a_1,a_2,a_3) and (b_1,b_2,b_3). respectively then the sum $\vec{A}+\vec{B}$ will have components $(a_1+b_1, a_2+b_2, a_3+b_3)$. It can be done diagramatically too by drawing the two vectors head to tail. The sum is the vector represented by the line segment from the tail of the first to the head of the second.

Vectorial Angle. See *Polar Coordinates.*

Velocities, Parallelogram of. See *Parallelogram of Vectors.*

Velocities, Triangle of. See *Triangle of Vectors.*

Velocity. Rate of change of displacement with respect to time for motion in line. Velocity is a vector quantity while speed is its magnitude. Velocity of a point with displacement x is dx/dt at any time t, in case of rectilinear motion. When the displacement is curvilinear that is, along a curve, the velocity is in the direction of the tangent to the curve at that point.

Velocity of light. Denoted usually by c. It is approximatly 3×10^5km/sec. See *Light Year.*

Venn Diagram. A schematic diagram used to show the relationship between the sets. Different sets are shown as circles in a big rectangle which denotes the universal set. Intersecting or overlapping circles denote intersecting sets. Separate circles show disjoint sets. A circle within another circle denotes a subset of a set.

Vertex, Vertices. (1) A point where lines or planes meet in a figure. For example, top point of a cone or a pyramid, point or points of intersection of a conic with its axis. (2) The intersection of the arms of an angle. (3) Intersecton of the sides of a polygon . (4) Intersection of the edges of a polyhedron.

Vertical Line. A line in a direction towards or away from the centre of the earth. Compare *Horizontal line.*

Vertically Opposite Angles. One of the two pairs of equal angles when two straight lines intersect each other.

Virtual Work. The work done if a system is displaced infinitesimally from its position. The virtual work is zero if the system is in equilibrium.

Visual Display Unit (VDU). A computer terminal by which the user can directly communicate with the computer. In the terminal there is a key board similar to that of a typewriter and the input and output appears on a television type screen.

Vital Statistics. Statistics dealing with the data on death, birth, marriages etc.

Volt. Unit of electromotive force (E.M.F.) and of potential difference. Unit volt is the potential difference between two points in a circuit through which one ampere current flows.

Volume. The measure of the space occupied by a solid or bounded by a closed surface. For example, the volume of a parallelopiped is the product of its length, breadth, and height.

Vulgar Fraction. Syn. *Common Fraction.*

W

Wallis' Product. $\frac{\pi}{2}=\frac{2.2.4.4.6.6.8.8.\ldots}{1.3.3.5.5.7.7.\ldots}$

Wave Equation. The partial differential equation

$$\frac{\partial^2 y}{\partial t^2}=c^2\frac{\partial^2 y}{\partial x^2}.$$

Weddle's Rule. The rule states

$$\int_a^b f(x)\,dx=\frac{3h}{10}\left[y_0+5y_1+y_2+6y_3+y_4+5y_5+2y_6+5y_7+y_8+\cdots\right]$$

where $h=\frac{b-a}{n}$, $y_0, y_1, y_2, \ldots$ are the values of $f(x)$ at $x=a, a+h, a+2h$ etc.

Wedge. The physical form of a dihedral angle. It is a device for causing a force in one direction to be effective in another. The principle involved is the resolution of forces.

Weight It is the force of attraction of the earth on any object near its surface. It is given by $w=mg$, where w is the weight, m the mass of the object and g the acceleration due to gravity. Due to variation of g from place to place, the weight also varies, whereas the mass remains constant.

Weighted Average (Mean). It is the mean of a set of numbers, each number having been associated with weights in the form of relative numerical importance. For example, if a students obtains 68, 83, 95 marks in the subjects English, Science and Mathematics respectively, and if the relative

importance, of these subjects are in the ratio of 2 : 3 : 4, called the weights, the weighted average (mean) will be

$$(2\times68+3\times83+4\times95)/(2+3+4)=85 \text{ marks.}$$

Whole Number. A positive integer or a *natural number*, *i.e.*, one of the numbers 1, 2, 3, 4, 5....

Width. Syn. *Breadth.*

Weierstras's Theorem. The theorem states that if a sequence of holomorphic functions converges uniformly in a domain then the limit function will also be holomorphic in the same domain.

Well Ordered Set. A partially ordered set is said to be well ordered if every subset of it contains first element. For example, the set of natural numbers with natural order is well ordered.

Wilson's Theorem. The necessary and sufficient condition for the number $(n-1)!+1$ to be divisible by n is that n is a prime number. Its necessary part was proved by Leibnitz while the sufficient part was proved by Lagrange. The number $(n-1)!+1$ is known as **Wilson's number.**

Witch of Agnesi This is a curve whose cartesian equation is $4a^2(2a-y)=x^2y$. Its parametric form,

$$x=2a\cot\theta,\ y=2a\sin^2\theta$$

is some times more convinient. For the shape of the curve, take two parallel lines at a distance $2a$ apart. Draw a circle which touches these lines. Let one point of contact be taken as origin of coordinates, O, the tangent at O as the x-axis and the diameter through O as y-axis. Draw any line through O cutting the circle at A and the parallel tangent at B. Draw a line through A parallel to x-axis and a line through B parallel to y-axis and let P be their point of intersection. The locus of P, *i.e.*, totality of all possible points P is the required curve called Witch of Agnesi given by Maria Agnesi in 1748. One important property of this curve is that the area between the curve and the x-axis is four times that of the generating circle.

Work. If a body A exerts a force F on another body B the energy is transferred from A to B and *work done* is measured by the scaler product $\vec{F} \cdot \vec{r}$, where $\vec{r}$ is the displacement produced in the direction of F. If the force varies along path C, then w the work done is given by

$$w = \int_C \vec{F} \cdot d\vec{r} \quad \text{where} \quad d\vec{r} = i\,dx + j\,dy + k\,dz$$

Wronskian. This is a determinant obtained while testing for the linear independence of a set of solutions of second order differential equations. Let f_i represent a set of functions then the criterion for linear *dependence* is the existence of a relation of the form $\Sigma k_i f_i = 0$, in which not all of the coefficients k_i are zero. On the other hand, if the only solution of the above equation is $k_i = 0$ for all i, the set of funclions f_i is said to be linearly independent. Assuming that the functions f_i are differentiable then, differentiating the above equation repeatedly we get a set of equations.

$$\Sigma k_i f_i' = 0 \qquad \Sigma k_i f_i'' = 0, \text{ etc.}$$

Thus, we have set of homogeneous linear equations in which k_i are unknowns. Now there is a solution $k_i = 0$, only if the determinant of the coefficients of the k_i's vanishes, that is

$$\begin{vmatrix} f_1 & f_2 & \cdots & f_n \\ f_1' & f_2' & \cdots & f_n' \\ f_1^{(n-1)} & f_2^{(n-1)} & \cdots & f_n^{(n-1)} \end{vmatrix} = 0$$

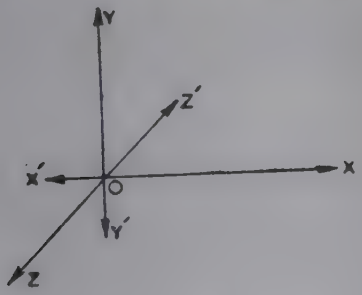
This determinant is called *wronskian.*

X

X. A roman symbol for the number 10.

X-, Y-, Z-Axes. Syn. *Coordinate Axes.* The three axes in the three dimension cartesian coordinate geometry. In the diagram the line $X'OX$, $Y'OY$, $Z'OZ$ through the origin of cordinates represent the X-,Y-,Z-axes. Their equations are,

$$y=0,\ z=0\ ;\quad z=0,\ x=0\ ;\quad x=0,\ y=0,$$



respectively.

x, y, z, Coordinates. Syn. *Cartesian Coordinates.* In the cartesian coordinates system x, y, z, are the distances measured upto the coordinate planes from the given point in space.

x, y, z, Intercepts. In the system of cartesian coordinates the distances from the origin to the points of intersection of the curve on the surface with x-axis, y-axis, z-axis respectively.

xy, zy, zx-Planes. Syn. *Coordinate Planes.* In the cartesian coordinate system the planes which contain the x-axis and y-axis, y-axis, z-axis, and z-axis and x-axis, are the xy, yz, and zx-planes respectively. Their equations are $z=0$, $x=0$, $y=0$ respectively.

Y

Y-Axis. See *X, Y, Z-Axes.*

y-Coordinate. See *x, y, z Coordinates.*

y-Intercept. See *x, y, z Intercepts.*

YZ-Plane. See *XY, YZ, ZX-Planes.*

Yard. The British Imperial and U.S. standard unit length. It is divided into 3 feet or 36 inches. It is also o thousand seven hundred-and sixteenth part of a statute *mi* This has now become obseletein science and engineering. O yard $\approx$ .9144 metres.

Year. The period of revolution of the earth around t sun. The *tropical* or the *Solar* year is the time that elaps between the two successive appearances at the first point Aries. It equals 365.2422 mean solar days.

The, Calendar year, is taken equivalent to 365.2422 me solar days.

The Sidereal year is taken to be 365.2564 mean solar da A lunar year, is equal to 12 lunar months and is taken to 365.3671 mean solar days. Three consecutive calender yea consist of 365 days and the fourth, known as a *leap* year, h 366 days. A year divisible by 4 is the leap year.

Young's Modulus. In a wire or a rod, the ratio longitudinal stress to longitudinal strain. It's unit is Newton/ or N/m^2.

Z

Z-Axis. See *X, Y, Z-Axis.*

z-Coordinates. See *x, y, z Coordinates.*

z-Intercept. See *x, y, z Intercepts.*

ZX-Plane. See *XY-*, YZ-, *ZX-Planes.*

Zenith. A point on the celestial sphere (or in the sky) directly overhead the observer. The highest point.

Zenith-Nadir Line. The line joining the *Zenith* to the *Nadir.*

Zero. It has the symbol 0 also called cipher, or naught which can be interpreted in the following several ways :

1. A *place holder* used in arthmetical computation.

2. A *cardinal number* associated with an empty set (a null set).

3. A point on the number line, dividing the positive and negative numbers.

4. One of the equal coordinates (0, 0, 0) of the origin in *cartesian coordinates.*

5. The limit of $1/n$ as n tends to infinity.

6. The power index which converts any number into unity.

7. The *Identity Element* with respect to addition of real numbers, *e.g* , $a+0=0=+a$.

8. The only number which can be divided by any non-zero number, the quotient always being zero.

9. The only number which cannot be a divisior.

10. The freezing-point of water on the Centigrade scale.

Zodiac. Zone of the celestial sphere in which the s
moon, and planets trace their paths. It is divided into
sections according to the apparent position of the sun in
12 months of the year. These sections are designated by
sign of the Zodiac : Aries (Rain), Tarus (Bull), Gemini (Twi
Cancer (Crab), Leo (Lion), Virgo (Virgin), Libra (Sca
Scorpio, (Scorpion), Sagitarius (Archer), Capricornus (
goat), Aquarius (Water—bearer), Pisces (Fishes).

Zone of Sphere. See spherical Cap, Spherical Zone.

(xii) $\frac{d}{dx}(\cot^{-1} x) = -\frac{1}{1+x^2}$

(xiii) $\frac{d}{dx}(\sec^{-1} x) = \frac{1}{x\sqrt{x^2-1}}$

(xiv) $\frac{d}{dx}(\operatorname{cosec}^{-1} x) = -\frac{1}{x\sqrt{x^2-1}}$

(xv) $\frac{d}{dx}(e^x) = e^x$

(xvi) $\frac{d}{dx}(a^x) = a^x \log_e a$

(xvii) $\frac{d}{dx}(\log_e x) = \frac{1}{x}$

(xviii) $\frac{d}{dx}(\log_a x) = \frac{1}{x} \log_e a$

(xix) $\frac{d}{dx}(\cosh x) = \sinh x$

(xx) $\frac{d}{dx}(\sinh x) = \cosh x$

(B)

(i) $\frac{d}{dx}[f(ax+b)] = a\ f'(ax+b)$, where f' stands for derivative of f, w.r.t.x

(ii) $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$

(iii) $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

(iv) If $y = f(t)$ and $t = \phi(x)$, then

$$\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx}$$

(v) $\frac{dy}{dx} = \frac{1}{\left(\frac{dx}{dy}\right)}$

APPENDIX I

STANDARD RESULTS (INTEGRAL CALCULUS)

The symbol $\int \ldots dx$. Here a, b, c are constants.

(A)

(*i*) $\int 0\, dx = c$

(*ii*) $\int x^n dx = \frac{x^{n+1}}{(n+1)} + c \quad (n \neq -1)$

(*iii*) $\int \cos x\, dx = \sin x + c$

(*iv*) $\int \sin x\, dx = -\cos x + c$

(*v*) $\int \sec^2 x dx = \tan x + c$

(*vi*) $\int \operatorname{cosec}^2 x\, dx = -\cot x + c$

(*vii*) $\int \sec x \tan x\, dx = \sec x + c$

(*viii*) $\int \operatorname{cosec} x \cot x\, dx = -\operatorname{cosec} x + c$

(*ix*) $\int \tan x\, dx = \log \sec x + c$

(*x*) $\int \cot x\, dx = \log \sin x + c$

(*xi*) $\int \sec x\, dx = \log (\sec x + \tan x) + c$

(*xii*) $\int \operatorname{cosec} x\, dx = -\log (\operatorname{cosec} x + \cot x) + c$

(*xiii*) $\int \frac{1}{x} dx = \log_e x + c$

(xiv) $\int e^x\,dx = e^x + c$

(xv) $\int a^x\,dx = a^x / \log_e a$

(xvi) $\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + c$

(xvii) $\int \frac{dx}{x\sqrt{x^2 - a^2}} = \frac{1}{a}\sec^{-1}\left(\frac{x}{a}\right) + c$

(xviii) $\int \frac{dx}{x^2 + a^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + c$

(xix) $\int \frac{dx}{x^2 - a^2} = \frac{1}{2a}\log\left(\frac{x-a}{x+a}\right) + c$

(xx) $\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \log\,(x + \sqrt{x^2 \pm a^2}) + c$

(xxi) $\int \sqrt{a^2 - x^2}\;dx = \frac{x\sqrt{a^2 - x^2}}{2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right) + c$

(xxii) $\int \sqrt{x^2 \pm a^2} = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} + a^2 \log\,(x + \sqrt{x^2 + a^2})\right] + c$

(xxiii) $\int\left[a f(x) + b\,\phi(x)\right]dx = a\int f(x)\,dx + b\int \phi(x)\,dx$

(xxiv) If $\int f(x)\,dx = \phi(x) + c$,

then $\int f(ax+b)dx = \frac{1}{a}\phi(ax+b) + c$

(xxv) If $\int f(x)\,dx = \phi(x) + c$, then $\int_a^b f(x)\,dx = \phi\,(b) - \phi\,(a)$

(xxvi) $\int u\,.\,v\,dx = u\,.\int v dx - \int \frac{du}{dx}\left(\int v\,dx\right)dx$

(B) Properties of Definite Integrals.

(*i*) $\int_a^b f(x)\,dx = \int_a^b f(t)\,dt$

(*ii*) $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$

(*iii*) $\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$

$a < c < b$

(*iv*) $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$

(*v*) If $f(x)$ is an odd function, then $\int_{-a}^{a} f(x)\,dx = 0$

(*vi*) If $f(x)$ is an even function

$$\int_{-a}^{a} f(x)\,dx = 2\int_0^a f(x)\,dx$$

(*vii*) If $f(2a-x) = -f(x)$, then $\int_0^{2a} f(x)\,dx = 0$

(*viii*) $$\int_0^{\pi/2} \sin^m x \cos^n x\,dx = \frac{\left|\overline{\frac{m+1}{2}}\right| \left|\overline{\frac{n+1}{2}}\right|}{2\left|\overline{\frac{m+n+2}{2}}\right|}$$

	0	1	2	3	4	5	6	7	8	9	1 2 3	4 5 6	7 8 9
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374	4 9 13	17 21 26	30 34 38
											4 8 12	16 20 24	28 32 36
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755	4 8 12	15 19 23	27 31 35
											4 7 11	15 19 22	26 30 33
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106	3 7 11	14 18 21	25 28 32
											3 7 10	14 17 20	24 27 31
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430	3 7 10	13 16 20	23 26 30
											3 7 10	13 16 19	22 25 29
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732	3 6 9	12 15 19	22 25 28
											3 6 9	12 15 17	20 23 26
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014	3 6 9	11 14 17	20 23 26
											3 6 8	11 14 17	19 22 24
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279	3 5 8	11 14 16	19 22 24
											3 5 8	10 13 16	18 21 23
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529	3 5 8	10 13 15	18 20 23
											2 5 7	10 12 15	17 20 22
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765	2 5 7	9 12 14	16 19 21
											2 5 7	9 11 14	16 18 21
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989	2 4 7	9 11 13	16 18 20
											2 4 6	8 11 13	15 17 19
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	2 4 6	8 11 13	15 17 19
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	2 4 6	8 10 12	14 16 18
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	2 4 6	8 10 12	14 15 17
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784	2 4 6	7 9 11	13 15 17
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	2 4 5	7 9 11	12 14 16
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	2 3 5	7 9 10	12 14 15
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	2 3 5	7 8 10	11 13 15
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	2 3 5	6 8 9	11 13 14
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	2 3 5	6 8 9	11 12 14
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757	1 3 4	6 7 9	10 12 13
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900	1 3 4	6 7 9	10 11 13
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038	1 3 4	6 7 8	10 11 12
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172	1 3 4	5 7 8	9 11 12
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	1 3 4	5 6 8	9 10 12
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428	1 3 4	5 6 8	9 10 11
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	1 2 4	5 6 7	9 10 11
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670	1 2 4	5 6 7	8 10 11
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	1 2 3	5 6 7	8 9 10
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	1 2 3	5 6 7	8 9 10
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	1 2 3	4 5 7	8 9 10
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117	1 2 3	4 5 6	8 9 10
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	1 2 3	4 5 6	7 8 9
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325	1 2 3	4 5 6	7 8 9
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44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522	1 2 3	4 5 6	7 8 9
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618	1 2 3	4 5 6	7 8 9
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712	1 2 3	4 5 6	7 7 8
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	1 2 3	4 5 5	6 7 8
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	1 2 3	4 4 5	6 7 8
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981	1 2 3	4 4 5	6 7 8

	0	1	2	3	4	5	6	7	8	9	1 2 3	4 5 6	7 8
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067	1 2 3	3 4 5	6 7
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152	1 2 3	3 4 5	6 7
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235	1 2 2	3 4 5	6 7
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316	1 2 2	3 4 5	6 6
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396	1 2 2	3 4 5	6 6
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474	1 2 2	3 4 5	5 6
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551	1 2 2	3 4 5	5 6
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627	1 2 2	3 4 5	5 6
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701	1 1 2	3 4 4	5 6
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774	1 1 2	3 4 4	5 6
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846	1 1 2	3 4 4	5 6
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917	1 1 2	3 4 4	5 6
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987	1 1 2	3 3 4	5 6
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055	1 1 2	3 3 4	5 5
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122	1 1 2	3 3 4	5 5
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189	1 1 2	3 3 4	5 5
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254	1 1 2	3 3 4	5 5
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319	1 1 2	3 3 4	5 5
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382	1 1 2	3 3 4	4 5
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445	1 1 2	2 3 4	4 5
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506	1 1 2	2 3 4	4 5
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73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686	1 1 2	2 3 4	4 5
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745	1 1 2	2 3 4	4 5
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802	1 1 2	2 3 3	4 5
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79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025	1 1 2	2 3 3	4 4
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83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238	1 1 2	2 3 3	4 4
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92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680	0 1 1	2 2 3	3 4
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97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908	0 1 1	2 2 3	3 4
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952	0 1 1	2 2 3	3 4
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996	0 1 1	2 2 3	3 4

	0	1	2	3	4	5	6	7	8	9	123	456	789
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·01	1023	1026	1028	1030	1033	1035	1038	1040	1042	1045	0 0 1	1 1 1	2 2 2
·02	1047	1050	1052	1054	1057	1059	1062	1064	1067	1069	0 0 1	1 1 1	2 2 2
·03	1072	1074	1076	1079	1081	1084	1086	1089	1091	1094	0 0 1	1 1 1	2 2 2
·04	1096	1099	1102	1104	1107	1109	1112	1114	1117	1119	0 1 1	1 1 2	2 2 2
·05	1122	1125	1127	1130	1132	1135	1138	1140	1143	1146	0 1 1	1 1 2	2 2 2
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·07	1175	1178	1180	1183	1186	1189	1191	1194	1197	1199	0 1 1	1 1 2	2 2 2
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·26	1820	1824	1828	1832	1837	1841	1845	1849	1854	1858	0 1 1	2 2 3	3 3 4
·27	1862	1866	1871	1875	1879	1884	1888	1892	1897	1901	0 1 1	2 2 3	3 3 4
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·31	2042	2046	2051	2056	2061	2065	2070	2075	2080	2084	0 1 1	2 2 3	3 4 4
·32	2089	2094	2099	2104	2109	2113	2118	2123	2128	2133	0 1 1	2 2 3	3 4 4
·33	2138	2143	2148	2153	2158	2163	2168	2173	2178	2183	0 1 1	2 2 3	3 4 4
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·35	2239	2244	2249	2254	2259	2265	2270	2275	2280	2286	1 1 2	2 3 3	4 4 5
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·37	2344	2350	2355	2360	2366	2371	2377	2382	2388	2393	1 1 2	2 3 3	4 4 5
·38	2399	2404	2410	2415	2421	2427	2432	2438	2443	2449	1 1 2	2 3 3	4 4 5
·39	2455	2460	2466	2472	2477	2483	2489	2495	2500	2506	1 1 2	2 3 3	4 5 5
·40	2512	2518	2523	2529	2535	2541	2547	2553	2559	2564	1 1 2	2 3 4	4 5 5
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·42	2630	2636	2642	2649	2655	2661	2667	2673	2679	2685	1 1 2	2 3 4	4 5 6
·43	2692	2698	2704	2710	2716	2723	2729	2735	2742	2748	1 1 2	3 3 4	4 5 6
·44	2754	2761	2767	2773	2780	2786	2793	2799	2805	2812	1 1 2	3 3 4	4 5 6
·45	2818	2825	2831	2838	2844	2851	2858	2864	2871	2877	1 1 2	3 3 4	5 5 6
·46	2884	2891	2897	2904	2911	2917	2924	2931	2938	2944	1 1 2	3 3 4	5 5 6
·47	2951	2958	2965	2972	2979	2985	2992	2999	3006	3013	1 1 2	3 3 4	5 5 6
·48	3020	3027	3034	3041	3048	3055	3062	3069	3076	3083	1 1 2	3 4 4	5 6 6
·49	3090	3097	3105	3112	3119	3126	3133	3141	3148	3155	1 1 2	3 4 4	5 6 6

	0	1	2	3	4	5	6	7	8	9	1 2 3	4 5 6	7 8 9
·50	3162	3170	3177	3184	3192	3199	3206	3214	3221	3228	1 1 2	3 4 4	5 6 7
·51	3236	3243	3251	3258	3266	3273	3281	3289	3296	3304	1 2 2	3 4 5	5 6 7
·52	3311	3319	3327	3334	3342	3350	3357	3365	3373	3381	1 2 2	3 4 5	5 6 7
·53	3388	3396	3404	3412	3420	3428	3436	3443	3451	3459	1 2 2	3 4 5	6 6 7
·54	3467	3475	3483	3491	3499	3508	3516	3524	3532	3540	1 2 2	3 4 5	6 6 7
·55	3548	3556	3565	3573	3581	3589	3597	3606	3614	3622	1 2 2	3 4 5	6 7 7
·56	3631	3639	3648	3656	3664	3673	3681	3690	3698	3707	1 2 3	3 4 5	6 7 8
·57	3715	3724	3733	3741	3750	3758	3767	3776	3784	3793	1 2 3	3 4 5	6 7 8
·58	3802	3811	3819	3828	3837	3846	3855	3864	3873	3882	1 2 3	4 4 5	6 7 8
·59	3890	3899	3908	3917	3926	3936	3945	3954	3963	3972	1 2 3	4 5 5	6 7 8
·60	3981	3990	3999	4009	4018	4027	4036	4046	4055	4064	1 2 3	4 5 6	6 7 8
·61	4074	4083	4093	4102	4111	4121	4130	4140	4150	4159	1 2 3	4 5 6	7 8 9
·62	4169	4178	4188	4198	4207	4217	4227	4236	4246	4256	1 2 3	4 5 6	7 8 9
·63	4266	4276	4285	4295	4305	4315	4325	4335	4345	4355	1 2 3	4 5 6	7 8 9
·64	4365	4375	4385	4395	4406	4416	4426	4436	4446	4457	1 2 3	4 5 6	7 8 9
·65	4467	4477	4487	4498	4508	4519	4529	4539	4550	4560	1 2 3	4 5 6	7 8 9
·66	4571	4581	4592	4603	4613	4624	4634	4645	4656	4667	1 2 3	4 5 6	7 9 10
·67	4677	4688	4699	4710	4721	4732	4742	4753	4764	4775	1 2 3	4 5 7	8 9 10
·68	4786	4797	4808	4819	4831	4842	4853	4864	4875	4887	1 2 3	4 6 7	8 9 10
·69	4898	4909	4920	4932	4943	4955	4966	4977	4989	5000	1 2 3	5 6 7	8 9 10
·70	5012	5023	5035	5047	5058	5070	5082	5093	5105	5117	1 2 4	5 6 7	8 9 11
·71	5129	5140	5152	5164	5176	5188	5200	5212	5224	5236	1 2 4	5 6 7	8 10 11
·72	5248	5260	5272	5284	5297	5309	5321	5333	5346	5358	1 2 4	5 6 7	9 10 11
·73	5370	5383	5395	5408	5420	5433	5445	5458	5470	5483	1 3 4	5 6 8	9 10 11
·74	5495	5508	5521	5534	5546	5559	5572	5585	5598	5610	1 3 4	5 6 8	9 10 12
·75	5623	5636	5649	5662	5675	5689	5702	5715	5728	5741	1 3 4	5 7 8	9 10 12
·76	5754	5768	5781	5794	5808	5821	5834	5848	5861	5875	1 3 4	5 7 8	9 11 12
·77	5888	5902	5916	5929	5943	5957	5970	5984	5998	6012	1 3 4	5 7 8	10 11 12
·78	6026	6039	6053	6067	6081	6095	6109	6124	6138	6152	1 3 4	6 7 8	10 11 13
·79	6166	6180	6194	6209	6223	6237	6252	6266	6281	6295	1 3 4	6 7 9	10 11 13
·80	6310	6324	6339	6353	6368	6383	6397	6412	6427	6442	1 3 4	6 7 9	10 12 13
·81	6457	6471	6486	6501	6516	6531	6546	6561	6577	6592	2 3 5	6 8 9	11 12 14
·82	6607	6622	6637	6653	6668	6683	6699	6714	6730	6745	2 3 5	6 8 9	11 12 14
·83	6761	6776	6792	6808	6823	6839	6855	6871	6887	6902	2 3 5	6 8 9	11 13 14
·84	6918	6934	6950	6966	6982	6998	7015	7031	7047	7063	2 3 5	6 8 10	11 13 15
·85	7079	7096	7112	7129	7145	7161	7178	7194	7211	7228	2 3 5	7 8 10	12 13 15
·86	7244	7261	7278	7295	7311	7328	7345	7362	7379	7396	2 3 5	7 8 10	12 13 15
·87	7413	7430	7447	7464	7482	7499	7516	7534	7551	7568	2 3 5	7 9 10	12 14 16
·88	7586	7603	7621	7638	7656	7674	7691	7709	7727	7745	2 4 5	7 9 11	12 14 16
·89	7762	7780	7798	7816	7834	7852	7870	7889	7907	7925	2 4 5	7 9 11	13 14 16
·90	7943	7962	7980	7998	8017	8035	8054	8072	8091	8110	2 4 6	7 9 11	13 15 17
·91	8128	8147	8166	8185	8204	8222	8241	8260	8279	8299	2 4 6	8 9 11	13 15 17
·92	8318	8337	8356	8375	8395	8414	8433	8453	8472	8492	2 4 6	8 10 12	14 15 17
·93	8511	8531	8551	8570	8590	8610	8630	8650	8670	8690	2 4 6	8 10 12	14 16 18
·94	8710	8730	8750	8770	8790	8810	8831	8851	8872	8892	2 4 6	8 10 12	14 16 18
·95	8913	8933	8954	8974	8995	9016	9036	9057	9078	9099	2 4 6	8 10 12	15 17 19
·96	9120	9141	9162	9183	9204	9226	9247	9268	9290	9311	2 4 6	8 11 13	15 17 19
·97	9333	9354	9376	9397	9419	9441	9462	9484	9506	9528	2 4 7	9 11 13	15 17 20
·98	9550	9572	9594	9616	9638	9661	9683	9705	9727	9750	2 4 7	9 11 13	16 18 20
·99	9772	9795	9817	9840	9863	9886	9908	9931	9954	9977	2 5 7	9 11 14	16 18 20

NATURAL SINES

Degrees	0′ 0°·0	6′ 0°·1	12′ 0°·2	18′ 0°·3	24′ 0°·4	30′ 0°·5	36′ 0°·6	42′ 0°·7	48′ 0°·8	54′ 0°·9	Mean Differences 1 2 3	4 5
0	·0000	0017	0035	0052	0070	0087	0105	0122	0140	0157	3 6 9	12 15
1	·0175	0192	0209	0227	0244	0262	0279	0297	0314	0332	3 6 9	12 15
2	·0349	0366	0384	0401	0419	0436	0454	0471	0488	0506	3 6 9	12 15
3	·0523	0541	0558	0576	0593	0610	0628	0645	0663	0680	3 6 9	12 15
4	·0698	0715	0732	0750	0767	0785	0802	0819	0837	0854	3 6 9	12 15
5	·0872	0889	0906	0924	0941	0958	0976	0993	1011	1028	3 6 9	12 14
6	·1045	1063	1080	1097	1115	1132	1149	1167	1184	1201	3 6 9	12 14
7	·1219	1236	1253	1271	1288	1305	1323	1340	1357	1374	3 6 9	12 14
8	·1392	1409	1426	1444	1461	1478	1495	1513	1530	1547	3 6 9	12 14
9	·1564	1582	1599	1616	1633	1650	1668	1685	1702	1719	3 6 9	12 14
10	·1736	1754	1771	1788	1805	1822	1840	1857	1874	1891	3 6 9	12 14
11	·1908	1925	1942	1959	1977	1994	2011	2028	2045	2062	3 6 9	11 14
12	·2079	2096	2113	2130	2147	2164	2181	2198	2215	2232	3 6 9	11 14
13	·2250	2267	2284	2300	2317	2334	2351	2368	2385	2402	3 6 8	11 14
14	·2419	2436	2453	2470	2487	2504	2521	2538	2554	2571	3 6 8	11 14
15	·2588	2605	2622	2639	2656	2672	2689	2706	2723	2740	3 6 8	11 14
16	·2756	2773	2790	2807	2823	2840	2857	2874	2890	2907	3 6 8	11 14
17	·2924	2940	2957	2974	2990	3007	3024	3040	3057	3074	3 6 8	11 14
18	·3090	3107	3123	3140	3156	3173	3190	3206	3223	3239	3 6 8	11 14
19	·3256	3272	3289	3305	3322	3338	3355	3371	3387	3404	3 5 8	11 14
20	·3420	3437	3453	3469	3486	3502	3518	3535	3551	3567	3 5 8	11 14
21	·3584	3600	3616	3633	3649	3665	3681	3697	3714	3730	3 5 8	11 14
22	·3746	3762	3778	3795	3811	3827	3843	3859	3875	3891	3 5 8	11 14
23	·3907	3923	3939	3955	3971	3987	4003	4019	4035	4051	3 5 8	11 14
24	·4067	4083	4099	4115	4131	4147	4163	4179	4195	4210	3 5 8	11 13
25	·4226	4242	4258	4274	4289	4305	4321	4337	4352	4368	3 5 8	11 13
26	·4384	4399	4415	4431	4446	4462	4478	4493	4509	4524	3 5 8	10 13
27	·4540	4555	4571	4586	4602	4617	4633	4648	4664	4679	3 5 8	10 13
28	·4695	4710	4726	4741	4756	4772	4787	4802	4818	4833	3 5 8	10 13
29	·4848	4863	4879	4894	4909	4924	4939	4955	4970	4985	3 5 8	10 13
30	·5000	5015	5030	5045	5060	5075	5090	5105	5120	5135	3 5 8	10 13
31	·5150	5165	5180	5195	5210	5225	5240	5255	5270	5284	2 5 7	10 12
32	·5299	5314	5329	5344	5358	5373	5388	5402	5417	5432	2 5 7	10 12
33	·5446	5461	5476	5490	5505	5519	5534	5548	5563	5577	2 5 7	10 12
34	·5592	5606	5621	5635	5650	5664	5678	5693	5707	5721	2 5 7	10 12
35	·5736	5750	5764	5779	5793	5807	5821	5835	5850	5864	2 5 7	10 12
36	·5878	5892	5906	5920	5934	5948	5962	5976	5990	6004	2 5 7	9 12
37	·6018	6032	6046	6060	6074	6088	6101	6115	6129	6143	2 5 7	9 12
38	·6157	6170	6184	6198	6211	6225	6239	6252	6266	6280	2 5 7	9 11
39	·6293	6307	6320	6334	6347	6361	6374	6388	6401	6414	2 4 7	9 11
40	·6428	6441	6455	6468	6481	6494	6508	6521	6534	6547	2 4 7	9 11
41	·6561	6574	6587	6600	6613	6626	6639	6652	6665	6678	2 4 7	9 11
42	·6691	6704	6717	6730	6743	6756	6769	6782	6794	6807	2 4 6	9 11
43	·6820	6833	6845	6858	6871	6884	6896	6909	6921	6934	2 4 6	8 11
44	·6947	6959	6972	6984	6997	7009	7022	7034	7046	7059	2 4 6	8 10

Degrees	0′ 0°·0	6′ 0°·1	12′ 0°·2	18′ 0°·3	24′ 0°·4	30′ 0°·5	36′ 0°·6	42′ 0°·7	48′ 0°·8	54′ 0°·9	Mean Differences 1 2 3	4	5
45	·7071	7083	7096	7108	7120	7133	7145	7157	7169	7181	2 4 6	8	10
46	·7193	7206	7218	7230	7242	7254	7266	7278	7290	7302	2 4 6	8	10
47	·7314	7325	7337	7349	7361	7373	7385	7396	7408	7420	2 4 6	8	10
48	·7431	7443	7455	7466	7478	7490	7501	7513	7524	7536	2 4 6	8	10
49	·7547	7558	7570	7581	7593	7604	7615	7627	7638	7649	2 4 6	8	9
50	·7660	7672	7683	7694	7705	7716	7727	7738	7749	7760	2 4 6	7	9
51	·7771	7782	7793	7804	7815	7826	7837	7848	7859	7869	2 4 5	7	9
52	·7880	7891	7902	7912	7923	7934	7944	7955	7965	7976	2 4 5	7	9
53	·7986	7997	8007	8018	8028	8039	8049	8059	8070	8080	2 3 5	7	9
54	·8090	8100	8111	8121	8131	8141	8151	8161	8171	8181	2 3 5	7	8
55	·8192	8202	8211	8221	8231	8241	8251	8261	8271	8281	2 3 5	7	8
56	·8290	8300	8310	8320	8329	8339	8348	8358	8368	8377	2 3 5	6	8
57	·8387	8396	8406	8415	8425	8434	8443	8453	8462	8471	2 3 5	6	8
58	·8480	8490	8499	8508	8517	8526	8536	8545	8554	8563	2 3 5	6	8
59	·8572	8581	8590	8599	8607	8616	8625	8634	8643	8652	1 3 4	6	7
60	·8660	8669	8678	8686	8695	8704	8712	8721	8729	8738	1 3 4	6	7
61	·8746	8755	8763	8771	8780	8788	8796	8805	8813	8821	1 3 4	6	7
62	·8829	8838	8846	8854	8862	8870	8878	8886	8894	8902	1 3 4	5	7
63	·8910	8918	8926	8934	8942	8949	8957	8965	8973	8980	1 3 4	5	6
64	·8988	8996	9003	9011	9018	9026	9033	9041	9048	9056	1 3 4	5	6
65	·9063	9070	9078	9085	9092	9100	9107	9114	9121	9128	1 2 4	5	6
66	·9135	9143	9150	9157	9164	9171	9178	9184	9191	9198	1 2 3	5	6
67	·9205	9212	9219	9225	9232	9239	9245	9252	9259	9265	1 2 3	4	6
68	·9272	9278	9285	9291	9298	9304	9311	9317	9323	9330	1 2 3	4	5
69	·9336	9342	9348	9354	9361	9367	9373	9379	9385	9391	1 2 3	4	5
70	·9397	9403	9409	9415	9421	9426	9432	9438	9444	9449	1 2 3	4	5
71	·9455	9461	9466	9472	9478	9483	9489	9494	9500	9505	1 2 3	4	5
72	·9511	9516	9521	9527	9532	9537	9542	9548	9553	9558	1 2 3	3	4
73	·9563	9568	9573	9578	9583	9588	9593	9598	9603	9608	1 2 2	3	4
74	·9613	9617	9622	9627	9632	9636	9641	9646	9650	9655	1 2 2	3	4
75	·9659	9664	9668	9673	9677	9681	9686	9690	9694	9699	1 1 2	3	4
76	·9703	9707	9711	9715	9720	9724	9728	9732	9736	9740	1 1 2	3	3
77	·9744	9748	9751	9755	9759	9763	9767	9770	9774	9778	1 1 2	3	3
78	·9781	9785	9789	9792	9796	9799	9803	9806	9810	9813	1 1 2	2	3
79	·9816	9820	9823	9826	9829	9833	9836	9839	9842	9845	1 1 2	2	3
80	·9848	9851	9854	9857	9860	9863	9866	9869	9871	9874	0 1 1	2	2
81	·9877	9880	9882	9885	9888	9890	9893	9895	9898	9900	0 1 1	2	2
82	·9903	9905	9907	9910	9912	9914	9917	9919	9921	9923	0 1 1	2	2
83	·9925	9928	9930	9932	9934	9936	9938	9940	9942	9943	0 1 1	1	2
84	·9945	9947	9949	9951	9952	9954	9956	9957	9959	9960	0 1 1	1	2
85	·9962	9963	9965	9966	9968	9969	9971	9972	9973	9974	0 0 1	1	1
86	·9976	9977	9978	9979	9980	9981	9982	9983	9984	9985	0 0 1	1	1
87	·9986	9987	9988	9989	9990	9990	9991	9992	9993	9993	0 0 0	1	1
88	·9994	9995	9995	9996	9996	9997	9997	9997	9998	9998	0 0 0	0	0
89	·9998	9999	9999	9999	9999	1·000	1·000	1·000	1·000	1·000	0 0 0	0	0
90	1·000												

NATURAL COSINES

[*Numbers in difference columns to be subtracted, not added.*]

Degrees	0′ 0°·0	6′ 0°·1	12′ 0°·2	18′ 0°·3	24′ 0°·4	30′ 0°·5	36′ 0°·6	42′ 0°·7	48′ 0°·8	54′ 0°·9	Mean Differences 1 2 3	4	5
0	1·000	1·000	1·000	1·000	1·000	1·000	·9999	9999	9999	9999	0 0 0	0	0
1	·9998	9998	9998	9997	9997	9997	9996	9996	9995	9995	0 0 0	0	0
2	·9994	9993	9993	9992	9991	9990	9990	9989	9988	9987	0 0 0	1	1
3	·9986	9985	9984	9983	9982	9981	9980	9979	9978	9977	0 0 1	1	1
4	·9976	9974	9973	9972	9971	9969	9968	9966	9965	9963	0 0 1	1	1
5	·9962	9960	9959	9957	9956	9954	9952	9951	9949	9947	0 1 1	1	2
6	·9945	9943	9942	9940	9938	9936	9934	9932	9930	9928	0 1 1	1	2
7	·9925	9923	9921	9919	9917	9914	9912	9910	9907	9905	0 1 1	2	2
8	·9903	9900	9898	9895	9893	9890	9888	9885	9882	9880	0 1 1	2	2
9	·9877	9874	9871	9869	9866	9863	9860	9857	9854	9851	0 1 1	2	2
10	·9848	9845	9842	9839	9836	9833	9829	9826	9823	9820	1 1 2	2	3
11	·9816	9813	9810	9806	9803	9799	9796	9792	9789	9785	1 1 2	2	3
12	·9781	9778	9774	9770	9767	9763	9759	9755	9751	9748	1 1 2	3	3
13	·9744	9740	9736	9732	9728	9724	9720	9715	9711	9707	1 1 2	3	3
14	·9703	9699	9694	9690	9686	9681	9677	9673	9668	9664	1 1 2	3	4
15	·9659	9655	9650	9646	9641	9636	9632	9627	9622	9617	1 2 2	3	4
16	·9613	9608	9603	9598	9593	9588	9583	9578	9573	9568	1 2 2	3	4
17	·9563	9558	9553	9548	9542	9537	9532	9527	9521	9516	1 2 3	3	4
18	·9511	9505	9500	9494	9489	9483	9478	9472	9466	9461	1 2 3	4	5
19	·9455	9449	9444	9438	9432	9426	9421	9415	9409	9403	1 2 3	4	5
20	·9397	9391	9385	9379	9373	9367	9361	9354	9348	9342	1 2 3	4	5
21	·9336	9330	9323	9317	9311	9304	9298	9291	9285	9278	1 2 3	4	5
22	·9272	9265	9259	9252	9245	9239	9232	9225	9219	9212	1 2 3	4	6
23	·9205	9198	9191	9184	9178	9171	9164	9157	9150	9143	1 2 3	5	6
24	·9135	9128	9121	9114	9107	9100	9092	9085	9078	9070	1 2 4	5	6
25	·9063	9056	9048	9041	9033	9026	9018	9011	9003	8996	1 3 4	5	6
26	·8988	8980	8973	8965	8957	8949	8942	8934	8926	8918	1 3 4	5	6
27	·8910	8902	8894	8886	8878	8870	8862	8854	8846	8838	1 3 4	5	7
28	·8829	8821	8813	8805	8796	8788	8780	8771	8763	8755	1 3 4	6	7
29	·8746	8738	8729	8721	8712	8704	8695	8686	8678	8669	1 3 4	6	7
30	·8660	8652	8643	8634	8625	8616	8607	8599	8590	8581	1 3 4	6	7
31	·8572	8563	8554	8545	8536	8526	8517	8508	8499	8490	2 3 5	6	8
32	·8480	8471	8462	8453	8443	8434	8425	8415	8406	8396	2 3 5	6	8
33	·8387	8377	8368	8358	8348	8339	8329	8320	8310	8300	2 3 5	6	8
34	·8290	8281	8271	8261	8251	8241	8231	8221	8211	8202	2 3 5	7	8
35	·8192	8181	8171	8161	8151	8141	8131	8121	8111	8100	2 3 5	7	8
36	·8090	8080	8070	8059	8049	8039	8028	8018	8007	7997	2 3 5	7	9
37	·7986	7976	7965	7955	7944	7934	7923	7912	7902	7891	2 4 5	7	9
38	·7880	7869	7859	7848	7837	7826	7815	7804	7793	7782	2 4 5	7	9
39	·7771	7760	7749	7738	7727	7716	7705	7694	7683	7672	2 4 6	7	9
40	·7660	7649	7638	7627	7615	7604	7593	7581	7570	7559	2 4 6	8	9
41	·7547	7536	7524	7513	7501	7490	7478	7466	7455	7443	2 4 6	8	10
42	·7431	7420	7408	7396	7385	7373	7361	7349	7337	7325	2 4 6	8	10
43	·7314	7302	7290	7278	7266	7254	7242	7230	7218	7206	2 4 6	8	10
44	·7193	7181	7169	7157	7145	7133	7120	7108	7096	7083	2 4 6	8	10

ACC. No 5099
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6-6-86

NATURAL COSINES

[Numbers in difference columns to be subtracted, not added.]

Degrees	0′ 0°·0	6′ 0°·1	12′ 0°·2	18′ 0°·3	24′ 0°·4	30′ 0°·5	36′ 0°·6	42′ 0°·7	48′ 0°·8	54′ 0°·9	Mean Differences 1 2 3	4 5
45	·7071	7059	7046	7034	7022	7009	6997	6984	6972	6959	2 4 6	8 10
46	·6947	6934	6921	6909	6896	6884	6871	6858	6845	6833	2 4 6	8 11
47	·6820	6807	6794	6782	6769	6756	6743	6730	6717	6704	2 4 6	9 11
48	·6691	6678	6665	6652	6639	6626	6613	6600	6587	6574	2 4 7	9 11
49	·6561	6547	6534	6521	6508	6494	6481	6468	6455	6441	2 4 7	9 11
50	·6428	6414	6401	6388	6374	6361	6347	6334	6320	6307	2 4 7	9 11
51	·6293	6280	6266	6252	6239	6225	6211	6198	6184	6170	2 5 7	9 11
52	·6157	6143	6129	6115	6101	6088	6074	6060	6046	6032	2 5 7	9 12
53	·6018	6004	5990	5976	5962	5948	5934	5920	5906	5892	2 5 7	9 12
54	·5878	5864	5850	5835	5821	5807	5793	5779	5764	5750	2 5 7	9 12
55	·5736	5721	5707	5693	5678	5664	5650	5635	5621	5606	2 5 7	10 12
56	·5592	5577	5563	5548	5534	5519	5505	5490	5476	5461	2 5 7	10 12
57	·5446	5432	5417	5402	5388	5373	5358	5344	5329	5314	2 5 7	10 12
58	·5299	5284	5270	5255	5240	5225	5210	5195	5180	5165	2 5 7	10 12
59	·5150	5135	5120	5105	5090	5075	5060	5045	5030	5015	3 5 8	10 13
60	·5000	4985	4970	4955	4939	4924	4909	4894	4879	4863	3 5 8	10 13
61	·4848	4833	4818	4802	4787	4772	4756	4741	4726	4710	3 5 8	10 13
62	·4695	4679	4664	4648	4633	4617	4602	4586	4571	4555	3 5 8	10 13
63	·4540	4524	4509	4493	4478	4462	4446	4431	4415	4399	3 5 8	10 13
64	·4384	4368	4352	4337	4321	4305	4289	4274	4258	4242	3 5 8	11 13
65	·4226	4210	4195	4179	4163	4147	4131	4115	4099	4083	3 5 8	11 13
66	·4067	4051	4035	4019	4003	3987	3971	3955	3939	3923	3 5 8	11 14
67	·3907	3891	3875	3859	3843	3827	3811	3795	3778	3762	3 5 8	11 14
68	·3746	3730	3714	3697	3681	3665	3649	3633	3616	3600	3 5 8	11 14
69	·3584	3567	3551	3535	3518	3502	3486	3469	3453	3437	3 5 8	11 14
70	·3420	3404	3387	3371	3355	3338	3322	3305	3289	3272	3 5 8	11 14
71	·3256	3239	3223	3206	3190	3173	3156	3140	3123	3107	3 6 8	11 14
72	·3090	3074	3057	3040	3024	3007	2990	2974	2957	2940	3 6 8	11 14
73	·2924	2907	2890	2874	2857	2840	2823	2807	2790	2773	3 6 8	11 14
74	·2756	2740	2723	2706	2689	2672	2656	2639	2622	2605	3 6 8	11 14
75	·2588	2571	2554	2538	2521	2504	2487	2470	2453	2436	3 6 8	11 14
76	·2419	2402	2385	2368	2351	2334	2317	2300	2284	2267	3 6 8	11 14
77	·2250	2233	2215	2198	2181	2164	2147	2130	2113	2096	3 6 9	11 14
78	·2079	2062	2045	2028	2011	1994	1977	1959	1942	1925	3 6 9	11 14
79	·1908	1891	1874	1857	1840	1822	1805	1788	1771	1754	3 6 9	11 14
80	·1736	1719	1702	1685	1668	1650	1633	1616	1599	1582	3 6 9	12 14
81	·1564	1547	1530	1513	1495	1478	1461	1444	1426	1409	3 6 9	12 14
82	·1392	1374	1357	1340	1323	1305	1288	1271	1253	1236	3 6 9	12 14
83	·1219	1201	1184	1167	1149	1132	1115	1097	1080	1063	3 6 9	12 14
84	·1045	1028	1011	0993	0976	0958	0941	0924	0906	0889	3 6 9	12 14
85	·0872	0854	0837	0819	0802	0785	0767	0750	0732	0715	3 6 9	12 15
86	·0698	0680	0663	0645	0628	0610	0593	0576	0558	0541	3 6 9	12 15
87	·0523	0506	0488	0471	0454	0436	0419	0401	0384	0366	3 6 9	12 15
88	·0349	0332	0314	0297	0279	0262	0244	0227	0209	0192	3 6 9	12 15
89	·0175	0157	0140	0122	0105	0087	0070	0052	0035	0017	3 6 9	12 15
90	·0000											

NATURAL TANGENTS

Degrees	0′ 0°·0	6′ 0°·1	12′ 0°·2	18′ 0°·3	24′ 0°·4	30′ 0°·5	36′ 0°·6	42′ 0°·7	48′ 0°·8	54′ 0°·9	Mean Differences 1	2	3	4	5
0	·0000	0017	0035	0052	0070	0087	0105	0122	0140	0157	3	6	9	12	15
1	·0175	0192	0209	0227	0244	0262	0279	0297	0314	0332	3	6	9	12	15
2	·0349	0367	0384	0402	0419	0437	0454	0472	0489	0507	3	6	9	12	15
3	·0524	0542	0559	0577	0594	0612	0629	0647	0664	0682	3	6	9	12	15
4	·0699	0717	0734	0752	0769	0787	0805	0822	0840	0857	3	6	9	12	15
5	·0875	0892	0910	0928	0945	0963	0981	0998	1016	1033	3	6	9	12	15
6	·1051	1069	1086	1104	1122	1139	1157	1175	1192	1210	3	6	9	12	15
7	·1228	1246	1263	1281	1299	1317	1334	1352	1370	1388	3	6	9	12	15
8	·1405	1423	1441	1459	1477	1495	1512	1530	1548	1566	3	6	9	12	15
9	·1584	1602	1620	1638	1655	1673	1691	1709	1727	1745	3	6	9	12	15
10	·1763	1781	1799	1817	1835	1853	1871	1890	1908	1926	3	6	9	12	15
11	·1944	1962	1980	1998	2016	2035	2053	2071	2089	2107	3	6	9	12	15
12	·2126	2144	2162	2180	2199	2217	2235	2254	2272	2290	3	6	9	12	15
13	·2309	2327	2345	2364	2382	2401	2419	2438	2456	2475	3	6	9	12	15
14	·2493	2512	2530	2549	2568	2586	2605	2623	2642	2661	3	6	9	12	16
15	·2679	2698	2717	2736	2754	2773	2792	2811	2830	2849	3	6	9	13	16
16	·2867	2886	2905	2924	2943	2962	2981	3000	3019	3038	3	6	9	13	16
17	·3057	3076	3096	3115	3134	3153	3172	3191	3211	3230	3	6	10	13	16
18	·3249	3269	3288	3307	3327	3346	3365	3385	3404	3424	3	6	10	13	16
19	·3443	3463	3482	3502	3522	3541	3561	3581	3600	3620	3	7	10	13	16
20	·3640	3659	3679	3699	3719	3739	3759	3779	3799	3819	3	7	10	13	17
21	·3839	3859	3879	3899	3919	3939	3959	3979	4000	4020	3	7	10	13	17
22	·4040	4061	4081	4101	4122	4142	4163	4183	4204	4224	3	7	10	14	17
23	·4245	4265	4286	4307	4327	4348	4369	4390	4411	4431	3	7	10	14	17
24	·4452	4473	4494	4515	4536	4557	4578	4599	4621	4642	4	7	11	14	18
25	·4663	4684	4706	4727	4748	4770	4791	4813	4834	4856	4	7	11	14	18
26	·4877	4899	4921	4942	4964	4986	5008	5029	5051	5073	4	7	11	15	18
27	·5095	5117	5139	5161	5184	5206	5228	5250	5272	5295	4	7	11	15	18
28	·5317	5340	5362	5384	5407	5430	5452	5475	5498	5520	4	8	11	15	19
29	·5543	5566	5589	5612	5635	5658	5681	5704	5727	5750	4	8	12	15	19
30	·5774	5797	5820	5844	5867	5890	5914	5938	5961	5985	4	8	12	16	20
31	·6009	6032	6056	6080	6104	6128	6152	6176	6200	6224	4	8	12	16	20
32	·6249	6273	6297	6322	6346	6371	6395	6420	6445	6469	4	8	12	16	20
33	·6494	6519	6544	6569	6594	6619	6644	6669	6694	6720	4	8	13	17	21
34	·6745	6771	6796	6822	6847	6873	6899	6924	6950	6976	4	9	13	17	21
35	·7002	7028	7054	7080	7107	7133	7159	7186	7212	7239	4	9	13	18	22
36	·7265	7292	7319	7346	7373	7400	7427	7454	7481	7508	5	9	14	18	23
37	·7536	7563	7590	7618	7646	7673	7701	7729	7757	7785	5	9	14	18	23
38	·7813	7841	7869	7898	7926	7954	7983	8012	8040	8069	5	9	14	19	24
39	·8098	8127	8156	8185	8214	8243	8273	8302	8332	8361	5	10	15	20	24
40	·8391	8421	8451	8481	8511	8541	8571	8601	8632	8662	5	10	15	20	25
41	·8693	8724	8754	8785	8816	8847	8878	8910	8941	8972	5	10	16	21	26
42	·9004	9036	9067	9099	9131	9163	9195	9228	9260	9293	5	11	16	21	27
43	·9325	9358	9391	9424	9457	9490	9523	9556	9590	9623	6	11	17	22	28
44	·9657	9691	9725	9759	9793	9827	9861	9896	9930	9965	6	11	17	23	29

NATURAL TANGENTS

Degrees	0′ 0°·0	6′ 0°·1	12′ 0°·2	18′ 0°·3	24′ 0°·4	30′ 0°·5	36′ 0°·6	42′ 0°·7	48′ 0°·8	54′ 0°·9	Mean Differences 1	2	3	4
45	1·0000	0035	0070	0105	0141	0176	0212	0247	0283	0319	6	12	18	24
46	1·0355	0392	0428	0464	0501	0538	0575	0612	0649	0686	6	12	18	25
47	1·0724	0761	0799	0837	0875	0913	0951	0990	1028	1067	6	13	19	25
48	1·1106	1145	1184	1224	1263	1303	1343	1383	1423	1463	7	13	20	27
49	1·1504	1544	1585	1626	1667	1708	1750	1792	1833	1875	7	14	21	28
50	1·1918	1960	2002	2045	2088	2131	2174	2218	2261	2305	7	14	22	29
51	1·2349	2393	2437	2482	2527	2572	2617	2662	2708	2753	8	15	23	30
52	1·2799	2846	2892	2938	2985	3032	3079	3127	3175	3222	8	16	24	31
53	1·3270	3319	3367	3416	3465	3514	3564	3613	3663	3713	8	16	25	33
54	1·3764	3814	3865	3916	3968	4019	4071	4124	4176	4229	9	17	26	34
55	1·4281	4335	4388	4442	4496	4550	4605	4659	4715	4770	9	18	27	36
56	1·4826	4882	4938	4994	5051	5108	5166	5224	5282	5340	10	19	29	38
57	1·5399	5458	5517	5577	5637	5697	5757	5818	5880	5941	10	20	30	40
58	1·6003	6066	6128	6191	6255	6319	6383	6447	6512	6577	11	21	32	43
59	1·6643	6709	6775	6842	6909	6977	7045	7113	7182	7251	11	23	34	45
60	1·7321	7391	7461	7532	7603	7675	7747	7820	7893	7966	12	24	36	48
61	1·8040	8115	8190	8265	8341	8418	8495	8572	8650	8728	13	26	38	51
62	1·8807	8887	8967	9047	9128	9210	9292	9375	9458	9542	14	27	41	55
63	1·9626	9711	9797	9883	9970	2·0057	2·0145	2·0233	2·0323	2·0413	15	29	44	58
64	2·0503	0594	0686	0778	0872	0965	1060	1155	1251	1348	16	31	47	63
65	2·1445	1543	1642	1742	1842	1943	2045	2148	2251	2355	17	34	51	68
66	2·2460	2566	2673	2781	2889	2998	3109	3220	3332	3445	18	37	55	73
67	2·3559	3673	3789	3906	4023	4142	4262	4383	4504	4627	20	40	60	79
68	2·4751	4876	5002	5129	5257	5386	5517	5649	5782	5916	22	43	65	87
69	2·6051	6187	6325	6464	6605	6746	6889	7034	7179	7326	24	47	71	95
70	2·7475	7625	7776	7929	8083	8239	8397	8556	8716	8878	26	52	78	104
71	2·9042	9208	9375	9544	9714	9887	3·0061	3·0237	3·0415	3·0595	29	58	87	116
72	3·0777	0961	1146	1334	1524	1716	1910	2106	2305	2506	32	64	96	129
73	3·2709	2914	3122	3332	3544	3759	3977	4197	4420	4646	36	72	108	144
74	3·4874	5105	5339	5576	5816	6059	6305	6554	6806	7062	41	81	122	163
75	3·7321	7583	7848	8118	8391	8667	8947	9232	9520	9812	46	93	139	186
76	4·0108	0408	0713	1022	1335	1653	1976	2303	2635	2972	53	107	160	213
77	4·3315	3662	4015	4374	4737	5107	5483	5864	6252	6646				
78	4·7046	7453	7867	8288	8716	9152	9594	5·0045	5·0504	5·0970				
79	5·1446	1929	2422	2924	3435	3955	4486	5026	5578	6140				
80	5·6713	7297	7894	8502	9124	9758	6·0405	6·1066	6·1742	6·2432				
81	6·3138	3859	4596	5350	6122	6912	7720	8548	9395	7·0264				
82	7·1154	2066	3002	3962	4947	5958	6996	8062	9158	8·0285				
83	8·1443	2636	3863	5126	6427	7769	9152	9·0579	9·2052	9·3572				
84	9·5144	9·677	9·845	10·02	10·20	10·39	10·58	10·78	10·99	11·20				
85	11·43	11·66	11·91	12·16	12·43	12·71	13·00	13·30	13·62	13·95				
86	14·30	14·67	15·06	15·46	15·89	16·35	16·83	17·34	17·89	18·46				
87	19·08	19·74	20·45	21·20	22·02	22·90	23·86	24·90	26·03	27·27				
88	28·64	30·14	31·82	33·69	35·80	38·19	40·92	44·07	47·74	52·08				
89	57·29	63·66	71·62	81·85	95·49	114·6	143·2	191·0	286·5	573·0				
90	∞													

Mean differences cea to be sufficient accurate.

NATURAL SECANTS.

Degrees.	0′ 0°·0	6′ 0°·1	12′ 0°·2	18′ 0°·3	24′ 0°·4	30′ 0°·5	36′ 0°·6	42′ 0°·7	48′ 0°·8	54′ 0°·9	Mean Differences. 1	2	3	4	5
0	1·0000	0000	0000	0000	0000	0000	0001	0001	0001	0001	0	0	0	0	0
1	1·0002	0002	0002	0003	0003	0003	0004	0004	0005	0006	0	0	0	0	0
2	1·0006	0007	0007	0008	0009	0010	0010	0011	0012	0013	0	0	0	0	0
3	1·0014	0015	0016	0017	0018	0019	0020	0021	0022	0023	0	0	1	1	1
4	1·0024	0026	0027	0028	0030	0031	0032	0034	0035	0037	0	0	1	1	1
5	1·0038	0040	0041	0043	0045	0046	0048	0050	0051	0053	0	1	1	1	1
6	1·0055	0057	0059	0061	0063	0065	0067	0069	0071	0073	0	1	1	1	2
7	1·0075	0077	0079	0082	0084	0086	0089	0091	0093	0096	0	1	1	2	2
8	1·0098	0101	0103	0106	0108	0111	0114	0116	0119	0122	0	1	1	2	2
9	1·0125	0127	0130	0133	0136	0139	0142	0145	0148	0151	0	1	1	2	2
10	1·0154	0157	0161	0164	0167	0170	0174	0177	0180	0184	1	1	2	2	3
11	1·0187	0191	0194	0198	0201	0205	0209	0212	0216	0220	1	1	2	2	3
12	1·0223	0227	0231	0235	0239	0243	0247	0251	0255	0259	1	1	2	3	3
13	1·0263	0267	0271	0276	0280	0284	0288	0293	0297	0302	1	1	2	3	4
14	1·0306	0311	0315	0320	0324	0329	0334	0338	0343	0348	1	2	2	3	4
15	1·0353	0358	0363	0367	0372	0377	0382	0388	0393	0398	1	2	3	3	4
16	1·0403	0408	0413	0419	0424	0429	0435	0440	0446	0451	1	2	3	3	4
17	1·0457	0463	0468	0474	0480	0485	0491	0497	0503	0509	1	2	3	4	5
18	1·0515	0521	0527	0533	0539	0545	0551	0557	0564	0570	1	2	3	4	5
19	1·0576	0583	0589	0595	0602	0608	0615	0622	0628	0635	1	2	3	4	5
20	1·0642	0649	0655	0662	0669	0676	0683	0690	0697	0704	1	2	3	5	6
21	1·0711	0719	0726	0733	0740	0748	0755	0763	0770	0778	1	2	4	5	6
22	1·0785	0793	0801	0808	0816	0824	0832	0840	0848	0856	1	3	4	5	6
23	1·0864	0872	0880	0888	0896	0904	0913	0921	0929	0938	1	3	4	6	7
24	1·0946	0955	0963	0972	0981	0989	0998	1007	1016	1025	1	3	4	6	7
25	1·1034	1043	1052	1061	1070	1079	1089	1098	1107	1117	2	3	5	6	8
26	1·1126	1136	1145	1155	1164	1174	1184	1194	1203	1213	2	3	5	6	8
27	1·1223	1233	1243	1253	1264	1274	1284	1294	1305	1315	2	3	5	7	9
28	1·1326	1336	1347	1357	1368	1379	1390	1401	1412	1423	2	4	5	7	9
29	1·1434	1445	1456	1467	1478	1490	1501	1512	1524	1535	2	4	6	8	9
30	1·1547	1559	1570	1582	1594	1606	1618	1630	1642	1654	2	4	6	8	10
31	1·1666	1679	1691	1703	1716	1728	1741	1753	1766	1779	2	4	6	8	10
32	1·1792	1805	1818	1831	1844	1857	1870	1883	1897	1910	2	4	7	9	11
33	1·1924	1937	1951	1964	1978	1992	2006	2020	2034	2048	2	5	7	9	12
34	1·2062	2076	2091	2105	2120	2134	2149	2163	2178	2193	2	5	7	10	12
35	1·2208	2223	2238	2253	2268	2283	2299	2314	2329	2345	3	5	8	10	13
36	1·2361	2376	2392	2408	2424	2440	2456	2472	2489	2505	3	5	8	11	13
37	1·2521	2538	2554	2571	2588	2605	2622	2639	2656	2673	3	6	8	11	14
38	1·2690	2708	2725	2742	2760	2778	2796	2813	2831	2849	3	6	9	12	15
39	1·2868	2886	2904	2923	2941	2960	2978	2997	3016	3035	3	6	9	12	16
40	1·3054	3073	3093	3112	3131	3151	3171	3190	3210	3230	3	7	10	13	16
41	1·3250	3270	3291	3311	3331	3352	3373	3393	3414	3435	3	7	10	14	17
42	1·3456	3478	3499	3520	3542	3563	3585	3607	3629	3651	4	7	11	14	18
43	1·3673	3696	3718	3741	3763	3786	3809	3832	3855	3878	4	8	11	15	19
44	1·3902	3925	3949	3972	3996	4020	4044	4069	4093	4118	4	8	12	16	20

Degrees.	0′ 0°·0	6′ 0°·1	12 0°·2	18′ 0°·3	24′ 0°·4	30′ 0°·5	36′ 0°·6	42′ 0°·7	48′ 0°·8	54′ 0°·9	Mean Differen			
											1	2	3	4
45	1·4142	4167	4192	4217	4242	4267	4293	4318	4344	4370	4	8	13	17
46	1·4396	4422	4448	4474	4501	4527	4554	4581	4608	4635	4	9	13	18
47	1·4663	4690	4718	4746	4774	4802	4830	4859	4887	4916	5	9	14	19
48	1·4945	4974	5003	5032	5062	5092	5121	5151	5182	5212	5	10	15	20
49	1·5243	5273	5304	5335	5366	5398	5429	5461	5493	5525	5	10	16	21
50	1·5557	5590	5622	5655	5688	5721	5755	5788	5822	5856	6	11	17	22
51	1·5890	5925	5959	5994	6029	6064	6099	6135	6171	6207	6	12	18	24
52	1·6243	6279	6316	6353	6390	6427	6464	6502	6540	6578	6	12	19	25
53	1·6616	6655	6694	6733	6772	6812	6852	6892	6932	6972	7	13	20	26
54	1·7013	7054	7095	7137	7179	7221	7263	7305	7348	7391	7	14	21	28
55	1·7434	7478	7522	7566	7610	7655	7700	7745	7791	7837	7	15	22	30
56	1·7883	7929	7976	8023	8070	8118	8166	8214	8263	8312	8	16	24	32
57	1·8361	8410	8460	8510	8561	8612	8663	8714	8766	8818	9	17	26	34
58	1·8871	8924	8977	9031	9084	9139	9194	9249	9304	9360	9	18	27	36
59	1·9416	9473	9530	9587	9645	9703	9762	9821	9880	9940	10	19	29	39
60	2·0000	0061	0122	0183	0245	0308	0371	0434	0498	0562	10	21	31	42
61	2·0627	0692	0757	0824	0890	0957	1025	1093	1162	1231	11	22	34	45
62	2 301	1371	1441	1513	1584	1657	1730	1803	1877	1952	12	24	36	48
63	2·2027	2103	2179	2256	2333	2412	2490	2570	2650	2730	13	26	39	52
64	2·2812	2894	2976	3060	3144	3228	3314	3400	3486	3574	14	28	43	57
65	2·3662	3751	3841	3931	4022	4114	4207	4300	4395	4490	15	31	46	62
66	2·4586	4683	4780	4879	4978	5078	5180	5282	5384	5488	17	34	50	67
67	2·5593	5699	5805	5913	6022	6131	6242	6354	6466	6580	18	37	55	73
68	2·6695	6811	6927	7046	7165	7285	7407	7529	7653	7778	20	40	60	81
69	2·7904	8032	8161	8291	8422	8555	8688	8824	8960	9099				
70	2·9238	9379	9521	9665	9811	9957	3·0106	3·0256	3·0407	3·0561				
71	3·0716	0872	1030	1190	1352	1515	1681	1848	2017	2188	Mean differer			
72	3·2361	2535	2712	2891	3072	3255	3440	3628	3817	4009	cease to be suffic			
73	3·4203	4399	4598	4799	5003	5209	5418	5629	5843	6060	accurate.			
74	3·6280	6502	6727	6955	7186	7420	7657	7897	8140	8387				
75	3·8637	8890	9147	9408	9672	9939	4·0211	4·0486	4·0765	4·1048				
76	4·1336	1627	1923	2223	2527	2837	3150	3469	3792	4121				
77	4·4454	4793	5137	5486	5841	6202	6569	6942	7321	7706				
78	4·8097	8496	8901	9313	9732	5·0159	5·0593	5·1034	5·1484	5·1942				
79	5·2408	2883	3367	3860	4362	4874	5396	5928	6470	7023				
80	5·7588	8164	8751	9351	9963	6·0589	6·1227	6·1880	6·2546	6·3228				
81	6·3925	4637	5366	6111	6874	7655	8454	9273	7·0112	7·0972				
82	7·1853	2757	3684	4635	5611	6613	7642	8700	9787	8·0905				
83	8·2055	3238	4457	5711	7004	8337	9711	9·1129	9·2593	9·4105				
84	9·5668	9·728	9·895	10·07	10·25	10·43	10·63	10·83	11·03	11·25				
85	11·47	11·71	11·95	12·20	12·47	12·75	13·03	13·34	13·65	13·99				
86	14·34	14·70	15·09	15·50	15·93	16·38	16·86	17·37	17·91	18·49				
87	19·11	19·77	20·47	21·23	22·04	22·93	23·88	24·92	26·05	27·29				
88	28·65	30·16	31·84	33·71	35·81	38·20	40·93	44·08	47·75	52·09				
89	57·30	63·66	71·62	81·85	95·49	114·6	143·2	191·0	286·5	573·0				

NATURAL COSECANTS.

[Numbers in difference columns to be subtracted, not added.]

	0′ 0°·0	6′ 0°·1	12′ 0°·2	18′ 0°·3	24′ 0°·4	30′ 0°·5	36′ 0°·6	42′ 0°·7	48′ 0°·8	54′ 0°·9	Mean Differences. 1	2	3	4	5
0	Inf.	573·0	286·5	191·0	143·2	114·6	95·49	81·85	71·62	63·66					
1	57·30	52·09	47·75	44·08	40·93	38·20	35·81	33·71	31·84	30·16					
2	28·65	27·29	26·05	24·92	23·88	22·93	22·04	21·23	20·47	19·77					
3	19·11	18·49	17·91	17·37	16·86	16·38	15·93	15·50	15·09	14·70					
4	14·34	13·99	13·65	13·34	13·03	12·75	12·47	12·20	11·95	11·71					
5	11·47	11·25	11·03	10·83	10·63	10·43	10·25	10·07	9·895	9·728					
6	9·5668	9·4105	9·2593	9·1129	8·9711	8·8337	8·7004	8·5711	8·4457	8·3238					
7	8·2055	8·0905	7·9787	7·8700	7·7642	7·6613	7·5611	7·4635	7·3684	7·2757	Mean differences are not sufficiently accurate.				
8	7·1853	7·0972	7·0112	6·9273	6·8454	6·7655	6·6874	6·6111	6·5366	6·4637					
9	6·3925	6·3228	6·2546	6·1880	6·1227	6·0589	5·9963	5·9351	5·8751	5·8164					
10	5·7588	5·7023	5·6470	5·5928	5·5396	5·4874	5·4362	5·3860	5·3367	5·2883					
11	5·2408	5·1942	5·1484	5·1034	5·0593	5·0150	4·9732	4·9313	4·8901	4·8496					
12	4·8097	4·7706	4·7321	4·6942	4·6569	4·6202	4·5841	4·5486	4·5137	4·4793					
13	4·4454	4·4121	4·3792	4·3469	4·3150	4·2837	4·2527	4·2223	4·1923	4·1627					
14	4·1336	4·1048	4·0765	4·0486	4·0211	3·9939	3·9672	3·9408	3·9147	3·8890					
15	3·8637	8387	8140	7897	7657	7420	7186	6955	6727	6502					
16	3·6280	6060	5843	5629	5418	5209	5003	4799	4598	4399					
17	3·4203	4009	3817	3628	3440	3255	3072	2891	2712	2535					
18	3·2361	2188	2017	1848	1681	1515	1352	1190	1030	0872	27	55	82	105	137
19	3·0716	0561	0407	0256	0106	2·9957	2·9811	2·9665	2·9521	2·9379	25	49	74	99	123
20	2·9238	9099	8960	8824	8688	8555	8422	8291	8161	8032	22	44	67	89	111
21	2·7904	7778	7653	7529	7407	7285	7165	7046	6927	6811	20	40	60	81	101
22	2·6695	6580	6466	6354	6242	6131	6022	5913	5805	5699	18	37	55	73	92
23	2·5593	5488	5384	5282	5180	5078	4978	4879	4780	4683	17	34	50	67	84
24	2·4586	4490	4395	4300	4207	4114	4022	3931	3841	3751	15	31	46	62	77
25	2·3662	3574	3486	3400	3314	3228	3144	3060	2976	2894	14	28	43	57	71
26	2·2812	2730	2650	2570	2490	2412	2333	2256	2179	2103	13	26	39	52	65
27	2·2027	1952	1877	1803	1730	1657	1584	1513	1441	1371	12	24	36	48	60
28	2·1301	1231	1162	1093	1025	0957	0890	0824	0757	0692	11	22	34	45	56
29	2·0627	0562	0498	0434	0371	0308	0245	0183	0122	0061	10	21	31	42	52
30	2·0000	1·9940	1·9880	1·9821	1·9762	1·9703	1·9645	1·9587	1·9530	1·9473	10	19	29	39	49
31	1·9416	9360	9304	9249	9194	9139	9084	9031	8977	8924	9	18	27	36	45
32	1·8871	8818	8766	8714	8663	8612	8561	8510	8460	8410	8	17	25	34	42
33	1·8361	8312	8263	8214	8166	8118	8070	8023	7976	7929	8	16	24	32	40
34	1·7883	7837	7791	7745	7700	7655	7610	7566	7522	7478	7	15	22	30	37
35	1·7434	7391	7348	7305	7263	7221	7179	7137	7095	7054	7	14	21	28	35
36	1·7013	6972	6932	6892	6852	6812	6772	6733	6694	6655	7	13	20	26	33
37	1·6616	6578	6540	6502	6464	6427	6390	6353	6316	6279	6	12	19	25	31
38	1·6243	6207	6171	6135	6099	6064	6029	5994	5959	5925	6	12	18	23	29
39	1·5890	5856	5822	5788	5755	5721	5688	5655	5622	5590	6	11	17	22	28
40	1·5557	5525	5493	5461	5429	5398	5366	5335	5304	5273	5	10	16	21	26
41	1·5243	5212	5182	5151	5121	5092	5062	5032	5003	4974	5	10	15	20	25
42	1·4945	4916	4887	4859	4830	4802	4774	4746	4718	4690	5	9	14	19	23
43	1·4663	4635	4608	4581	4554	4527	4501	4474	4448	4422	4	9	13	18	22
44	1·4396	4370	4344	4318	4293	4267	4242	4217	4192	4167	4	8	13	17	21

NATURAL COSECANTS.

[Numbers in difference columns to be subtracted, not added.]

Degrees.	0′ 0°·0	6′ 0°·1	12′ 0°·2	18′ 0°·3	24′ 0°·4	30′ 0°·5	36′ 0°·6	42′ 0°·7	48′ 0°·8	54′ 0°·9	Mean Differen			
											1	2	3	4
45	1·4142	4118	4093	4069	4044	4020	3996	3972	3949	3925	4	8	12	16
46	1·3902	3878	3855	3832	3809	3786	3763	3741	3718	3696	4	8	11	15
47	1·3673	3651	3629	3607	3585	3563	3542	3520	3499	3478	4	7	11	14
48	1·3456	3435	3414	3393	3373	3352	3331	3311	3291	3270	3	7	10	14
49	1·3250	3230	3210	3190	3171	3151	3131	3112	3093	3073	3	7	10	13
50	1·3054	3035	3016	2997	2978	2960	2941	2923	2904	2886	3	6	9	12
51	1·2868	2849	2831	2813	2796	2778	2760	2742	2725	2708	3	6	9	12
52	1·2690	2673	2656	2639	2622	2605	2588	2571	2554	2538	3	6	8	11
53	1·2521	2505	2489	2472	2456	2440	2424	2408	2392	2376	3	5	8	11
54	1·2361	2345	2329	2314	2299	2283	2268	2253	2238	2223	3	5	8	10
55	1·2208	2193	2178	2163	2149	2134	2120	2105	2091	2076	2	5	7	10
56	1·2062	2048	2034	2020	2006	1992	1978	1964	1951	1937	2	5	7	9
57	1·1924	1910	1897	1883	1870	1857	1844	1831	1818	1805	2	4	7	9
58	1·1792	1779	1766	1753	1741	1728	1716	1703	1691	1679	2	4	6	8
59	1·1666	1654	1642	1630	1618	1606	1594	1582	1570	1559	2	4	6	8
60	1·1547	1535	1524	1512	1501	1490	1478	1467	1456	1445	2	4	6	8
61	1·1434	1423	1412	1401	1390	1379	1368	1357	1347	1336	2	4	5	7
62	1·1326	1315	1305	1294	1284	1274	1264	1253	1243	1233	2	3	5	7
63	1·1223	1213	1203	1194	1184	1174	1164	1155	1145	1136	2	3	5	6
64	1·1126	1117	1107	1098	1089	1079	1070	1061	1052	1043	2	3	5	6
65	1·1034	1025	1016	1007	0998	0989	0981	0972	0963	0955	1	3	4	6
66	1·0946	0938	0929	0921	0913	0904	0896	0888	0880	0872	1	3	4	6
67	1·0864	0856	0848	0840	0832	0824	0816	0808	0801	0793	1	3	4	5
68	1·0785	0778	0770	0763	0755	0748	0740	0733	0726	0719	1	2	4	5
69	1·0711	0704	0697	0690	0683	0676	0669	0662	0655	0649	1	2	3	5
70	1·0642	0635	0628	0622	0615	0608	0602	0595	0589	0583	1	2	3	4
71	1·0576	0570	0564	0557	0551	0545	0539	0533	0527	0521	1	2	3	4
72	1·0515	0509	0503	0497	0491	0485	0480	0474	0468	0463	1	2	3	4
73	1·0457	0451	0446	0440	0435	0429	0424	0419	0413	0408	1	2	3	3
74	1·0403	0398	0393	0388	0382	0377	0372	0367	0363	0358	1	2	2	3
75	1·0353	0348	0343	0338	0334	0329	0324	0320	0315	0311	1	2	2	3
76	1·0306	0302	0297	0293	0288	0284	0280	0276	0271	0267	1	1	2	3
77	1·0263	0259	0255	0251	0247	0243	0239	0235	0231	0227	1	1	2	3
78	1·0223	0220	0216	0212	0209	0205	0201	0198	0194	0191	1	1	2	2
79	1·0187	0184	0180	0177	0174	0170	0167	0164	0161	0157	1	1	2	2
80	1·0154	0151	0148	0145	0142	0139	0136	0133	0130	0127	0	1	1	2
81	1·0125	0122	0119	0116	0114	0111	0108	0106	0103	0101	0	1	1	2
82	1·0098	0096	0093	0091	0089	0086	0084	0082	0079	0077	0	1	1	2
83	1·0075	0073	0071	0069	0067	0065	0063	0061	0059	0057	0	1	1	1
84	1·0055	0053	0051	0050	0048	0046	0045	0043	0041	0040	0	1	1	1
85	1·0038	0037	0035	0034	0032	0031	0030	0028	0027	0026	0	0	1	1
86	1·0024	0023	0022	0021	0020	0019	0018	0017	0016	0015	0	0	0	1
87	1·0014	0013	0012	0011	0010	0010	0009	0008	0007	0007	0	0	0	1
88	1·0006	0006	0005	0004	0004	0003	0003	0003	0002	0002	0	0	0	0
89	1·0002	0001	0001	0001	0001	0000	0000	0000	0000	0000	0	0	0	0

Degrees	0′ 0°·0	6′ 0°·1	12′ 0°·2	18′ 0°·3	24′ 0°·4	30′ 0°·5	36′ 0°·6	42′ 0°·7	48′ 0°·8	54′ 0°·9	Mean Differences 1 2 3	4 5
0	∞	$\bar{3}$·2419	$\bar{3}$·5429	7190	8439	9408	$\bar{2}$·0200	$\bar{2}$·0870	$\bar{2}$·1450	$\bar{2}$·1961		
1	$\bar{2}$·2419	2832	3210	3558	3880	4179	4459	4723	4971	5206		
2	$\bar{2}$·5428	5640	5842	6035	6220	6397	6567	6731	6889	7041		
3	$\bar{2}$·7188	7330	7468	7602	7731	7857	7979	8098	8213	8326		
4	$\bar{2}$·8436	8543	8647	8749	8849	8946	9042	9135	9226	9315	16 32 48	64 80
5	$\bar{2}$·9403	9489	9573	9655	9736	9816	9894	9970	$\bar{1}$·0046	$\bar{1}$·0120	13 26 39	52 65
6	$\bar{1}$·0192	0264	0334	0403	0472	0539	0605	0670	0734	0797	11 22 33	44 55
7	$\bar{1}$·0859	0920	0981	1040	1099	1157	1214	1271	1326	1381	10 19 29	38 48
8	$\bar{1}$·1436	1489	1542	1594	1646	1697	1747	1797	1847	1895	8 17 25	34 42
9	$\bar{1}$·1943	1991	2038	2085	2131	2176	2221	2266	2310	2353	8 15 23	30 38
10	$\bar{1}$·2397	2439	2482	2524	2565	2606	2647	2687	2727	2767	7 14 20	27 34
11	$\bar{1}$·2806	2845	2883	2921	2959	2997	3034	3070	3107	3143	6 12 19	25 31
12	$\bar{1}$·3179	3214	3250	3284	3319	3353	3387	3421	3455	3488	6 11 17	23 28
13	$\bar{1}$·3521	3554	3586	3618	3650	3682	3713	3745	3775	3806	5 11 16	21 26
14	$\bar{1}$·3837	3867	3897	3927	3957	3986	4015	4044	4073	4102	5 10 15	20 24
15	$\bar{1}$·4130	4158	4186	4214	4242	4269	4296	4323	4350	4377	5 9 14	18 23
16	$\bar{1}$·4403	4430	4456	4482	4508	4533	4559	4584	4609	4634	4 9 13	17 21
17	$\bar{1}$·4659	4684	4709	4733	4757	4781	4805	4829	4853	4876	4 8 12	16 20
18	$\bar{1}$·4900	4923	4946	4969	4992	5015	5037	5060	5082	5104	4 8 11	15 19
19	$\bar{1}$·5126	5148	5170	5192	5213	5235	5256	5278	5299	5320	4 7 11	14 18
20	$\bar{1}$·5341	5361	5382	5402	5423	5443	5463	5484	5504	5523	3 7 10	14 17
21	$\bar{1}$·5543	5563	5583	5602	5621	5641	5660	5679	5698	5717	3 6 10	13 16
22	$\bar{1}$·5736	5754	5773	5792	5810	5828	5847	5865	5883	5901	3 6 9	12 15
23	$\bar{1}$·5919	5937	5954	5972	5990	6007	6024	6042	6059	6076	3 6 9	12 15
24	$\bar{1}$·6093	6110	6127	6144	6161	6177	6194	6210	6227	6243	3 6 8	11 14
25	$\bar{1}$·6259	6276	6292	6308	6324	6340	6356	6371	6387	6403	3 5 8	11 13
26	$\bar{1}$·6418	6434	6449	6465	6480	6495	6510	6526	6541	6556	3 5 8	10 13
27	$\bar{1}$·6570	6585	6600	6615	6629	6644	6659	6673	6687	6702	2 5 7	10 12
28	$\bar{1}$·6716	6730	6744	6759	6773	6787	6801	6814	6828	6842	2 5 7	9 12
29	$\bar{1}$·6856	6869	6883	6896	6910	6923	6937	6950	6963	6977	2 4 7	9 11
30	$\bar{1}$·6990	7003	7016	7029	7042	7055	7068	7080	7093	7106	2 4 6	9 11
31	$\bar{1}$·7118	7131	7144	7156	7168	7181	7193	7205	7218	7230	2 4 6	8 10
32	$\bar{1}$·7242	7254	7266	7278	7290	7302	7314	7326	7338	7349	2 4 6	8 10
33	$\bar{1}$·7361	7373	7384	7396	7407	7419	7430	7442	7453	7464	2 4 6	8 10
34	$\bar{1}$·7476	7487	7498	7509	7520	7531	7542	7553	7564	7575	2 4 6	7 9
35	$\bar{1}$·7586	7597	7607	7618	7629	7640	7650	7661	7671	7682	2 4 5	7 9
36	$\bar{1}$·7692	7703	7713	7723	7734	7744	7754	7764	7774	7785	2 3 5	7 9
37	$\bar{1}$·7795	7805	7815	7825	7835	7844	7854	7864	7874	7884	2 3 5	7 8
38	$\bar{1}$·7893	7903	7913	7922	7932	7941	7951	7960	7970	7979	2 3 5	6 8
39	$\bar{1}$·7989	7998	8007	8017	8026	8035	8044	8053	8063	8072	2 3 5	6 8
40	$\bar{1}$·8081	8090	8099	8108	8117	8125	8134	8143	8152	8161	1 3 4	6 7
41	$\bar{1}$·8169	8178	8187	8195	8204	8213	8221	8230	8238	8247	1 3 4	6 7
42	$\bar{1}$·8255	8264	8272	8280	8289	8297	8305	8313	8322	8330	1 3 4	6 7
43	$\bar{1}$·8338	8346	8354	8362	8370	8378	8386	8394	8402	8410	1 3 4	5 7
44	$\bar{1}$·8418	8426	8433	8441	8449	8457	8464	8472	8480	8487	1 3 4	5 6

Degrees	0′ 0°·0	6′ 0°·1	12′ 0°·2	18′ 0°·3	24′ 0°·4	30′ 0°·5	36′ 0°·6	42′ 0°·7	48′ 0°·8	54′ 0°·9	Mean Differences 1	2	3	4	5
45	$\bar{1}$·8495	8502	8510	8517	8525	8532	8540	8547	8555	8562	1	2	4	5	6
46	$\bar{1}$·8569	8577	8584	8591	8598	8606	8613	8620	8627	8634	1	2	4	5	6
47	$\bar{1}$·8641	8648	8655	8662	8669	8676	8683	8690	8697	8704	1	2	3	5	6
48	$\bar{1}$·8711	8718	8724	8731	8738	8745	8751	8758	8765	8771	1	2	3	4	6
49	$\bar{1}$·8778	8784	8791	8797	8804	8810	8817	8823	8830	8836	1	2	3	4	5
50	$\bar{1}$·8843	8849	8855	8862	8868	8874	8880	8887	8893	8899	1	2	3	4	5
51	$\bar{1}$·8905	8911	8917	8923	8929	8935	8941	8947	8953	8959	1	2	3	4	5
52	$\bar{1}$·8965	8971	8977	8983	8989	8995	9000	9006	9012	9018	1	2	3	4	5
53	$\bar{1}$·9023	9029	9035	9041	9046	9052	9057	9063	9069	9074	1	2	3	4	5
54	$\bar{1}$·9080	9085	9091	9096	9101	9107	9112	9118	9123	9128	1	2	3	4	5
55	$\bar{1}$·9134	9139	9144	9149	9155	9160	9165	9170	9175	9181	1	2	3	3	4
56	$\bar{1}$·9186	9191	9196	9201	9206	9211	9216	9221	9226	9231	1	2	3	3	4
57	$\bar{1}$·9236	9241	9246	9251	9255	9260	9265	9270	9275	9279	1	2	2	3	4
58	$\bar{1}$·9284	9289	9294	9298	9303	9308	9312	9317	9322	9326	1	2	2	3	4
59	$\bar{1}$·9331	9335	9340	9344	9349	9353	9358	9362	9367	9371	1	1	2	3	4
60	$\bar{1}$·9375	9380	9384	9388	9393	9397	9401	9406	9410	9414	1	1	2	3	4
61	$\bar{1}$·9418	9422	9427	9431	9435	9439	9443	9447	9451	9455	1	1	2	3	3
62	$\bar{1}$·9459	9463	9467	9471	9475	9479	9483	9487	9491	9495	1	1	2	3	3
63	$\bar{1}$·9499	9503	9506	9510	9514	9518	9522	9525	9529	9533	1	1	2	3	3
64	$\bar{1}$·9537	9540	9544	9548	9551	9555	9558	9562	9566	9569	1	1	2	2	3
65	$\bar{1}$·9573	9576	9580	9583	9587	9590	9594	9597	9601	9604	1	1	2	2	3
66	$\bar{1}$·9607	9611	9614	9617	9621	9624	9627	9631	9634	9637	1	1	2	2	3
67	$\bar{1}$·9640	9643	9647	9650	9653	9656	9659	9662	9666	9669	1	1	2	2	3
68	$\bar{1}$·9672	9675	9678	9681	9684	9687	9690	9693	9696	9699	0	1	1	2	2
69	$\bar{1}$·9702	9704	9707	9710	9713	9716	9719	9722	9724	9727	0	1	1	2	2
70	$\bar{1}$·9730	9733	9735	9738	9741	9743	9746	9749	9751	9754	0	1	1	2	2
71	$\bar{1}$·9757	9759	9762	9764	9767	9770	9772	9775	9777	9780	0	1	1	2	2
72	$\bar{1}$·9782	9785	9787	9789	9792	9794	9797	9799	9801	9804	0	1	1	2	2
73	$\bar{1}$·9806	9808	9811	9813	9815	9817	9820	9822	9824	9826	0	1	1	2	2
74	$\bar{1}$·9828	9831	9833	9835	9837	9839	9841	9843	9845	9847	0	1	1	1	2
75	$\bar{1}$·9849	9851	9853	9855	9857	9859	9861	9863	9865	9867	0	1	1	1	2
76	$\bar{1}$·9869	9871	9873	9875	9876	9878	9880	9882	9884	9885	0	1	1	1	2
77	$\bar{1}$·9887	9889	9891	9892	9894	9896	9897	9899	9901	9902	0	1	1	1	1
78	$\bar{1}$·9904	9906	9907	9909	9910	9912	9913	9915	9916	9918	0	1	1	1	1
79	$\bar{1}$·9919	9921	9922	9924	9925	9927	9928	9929	9931	9932	0	0	1	1	1
80	$\bar{1}$·9934	9935	9936	9937	9939	9940	9941	9943	9944	9945	0	0	1	1	1
81	$\bar{1}$·9946	9947	9949	9950	9951	9952	9953	9954	9955	9956	0	0	1	1	1
82	$\bar{1}$·9958	9959	9960	9961	9962	9963	9964	9965	9966	9967	0	0	1	1	1
83	$\bar{1}$·9968	9968	9969	9970	9971	9972	9973	9974	9975	9975	0	0	0	1	1
84	$\bar{1}$·9976	9977	9978	9978	9979	9980	9981	9981	9982	9983	0	0	0	0	1
85	$\bar{1}$·9983	9984	9985	9985	9986	9987	9987	9988	9988	9989	0	0	0	0	0
86	$\bar{1}$·9989	9990	9990	9991	9991	9992	9992	9993	9993	9994	0	0	0	0	0
87	$\bar{1}$·9994	9994	9995	9995	9996	9996	9996	9996	9997	9997	0	0	0	0	0
88	$\bar{1}$·9997	9998	9998	9998	9998	9999	9999	9999	9999	9999	0	0	0	0	0
89	$\bar{1}$·9999	9999	0·0000	0000	0000	0000	0000	0000	0000	0000					
90	0·0000														

LOGARITHMS OF COSINES

[Numbers in difference columns to be subtracted, not added.]

Degrees	0′ 0°·0	6′ 0°·1	12′ 0°·2	18′ 0°·3	24′ 0°·4	30′ 0°·5	36′ 0°·6	42′ 0°·7	48′ 0°·8	54′ 0°·9	Mean Differences 1	2	3	4	5
0	0·0000	0000	0000	0000	0000	0000	0000	0000	0000	$\bar{1}$·9999	0	0	0	0	0
1	$\bar{1}$·9999	9999	9999	9999	9999	9999	9998	9998	9998	9998	0	0	0	0	0
2	$\bar{1}$·9997	9997	9997	9996	9996	9996	9996	9995	9995	9994	0	0	0	0	0
3	$\bar{1}$·9994	9994	9993	9993	9992	9992	9991	9991	9990	9990	0	0	0	0	0
4	$\bar{1}$·9989	9989	9988	9988	9987	9987	9986	9985	9985	9984	0	0	0	0	0
5	$\bar{1}$·9983	9983	9982	9981	9981	9980	9979	9978	9978	9977	0	0	0	0	1
6	$\bar{1}$·9976	9975	9975	9974	9973	9972	9971	9970	9969	9968	0	0	0	1	1
7	$\bar{1}$·9968	9967	9966	9965	9964	9963	9962	9961	9960	9959	0	0	1	1	1
8	$\bar{1}$·9958	9956	9955	9954	9953	9952	9951	9950	9949	9947	0	0	1	1	1
9	$\bar{1}$·9946	9945	9944	9943	9941	9940	9939	9937	9936	9935	0	0	1	1	1
10	$\bar{1}$·9934	9932	9931	9929	9928	9927	9925	9924	9922	9921	0	0	1	1	1
11	$\bar{1}$·9919	9918	9916	9915	9913	9912	9910	9909	9907	9906	0	1	1	1	1
12	$\bar{1}$·9904	9902	9901	9899	9897	9896	9894	9892	9891	9889	0	1	1	1	1
13	$\bar{1}$·9887	9885	9884	9882	9880	9878	9876	9875	9873	9871	0	1	1	1	2
14	$\bar{1}$·9869	9867	9865	9863	9861	9859	9857	9855	9853	9851	0	1	1	1	2
15	$\bar{1}$·9849	9847	9845	9843	9841	9839	9837	9835	9833	9831	0	1	1	1	2
16	$\bar{1}$·9828	9826	9824	9822	9820	9817	9815	9813	9811	9808	0	1	1	2	2
17	$\bar{1}$·9806	9804	9801	9799	9797	9794	9792	9789	9787	9785	0	1	1	2	2
18	$\bar{1}$·9782	9780	9777	9775	9772	9770	9767	9764	9762	9759	0	1	1	2	2
19	$\bar{1}$·9757	9754	9751	9749	9746	9743	9741	9738	9735	9733	0	1	1	2	2
20	$\bar{1}$·9730	9727	9724	9722	9719	9716	9713	9710	9707	9704	0	1	1	2	2
21	$\bar{1}$·9702	9699	9696	9693	9690	9687	9684	9681	9678	9675	0	1	1	2	2
22	$\bar{1}$·9672	9669	9666	9662	9659	9656	9653	9650	9647	9643	1	1	2	2	3
23	$\bar{1}$·9640	9637	9634	9631	9627	9624	9621	9617	9614	9611	1	1	2	2	3
24	$\bar{1}$·9607	9604	9601	9597	9594	9590	9587	9583	9580	9576	1	1	2	2	3
25	$\bar{1}$·9573	9569	9566	9562	9558	9555	9551	9548	9544	9540	1	1	2	2	3
26	$\bar{1}$·9537	9533	9529	9525	9522	9518	9514	9510	9506	9503	1	1	2	3	3
27	$\bar{1}$·9499	9495	9491	9487	9483	9479	9475	9471	9467	9463	1	1	2	3	3
28	$\bar{1}$·9459	9455	9451	9447	9443	9439	9435	9431	9427	9422	1	1	2	3	3
29	$\bar{1}$·9418	9414	9410	9406	9401	9397	9393	9388	9384	9380	1	1	2	3	4
30	$\bar{1}$·9375	9371	9367	9362	9358	9353	9349	9344	9340	9335	1	1	2	3	4
31	$\bar{1}$·9331	9326	9322	9317	9312	9308	9303	9298	9294	9289	1	2	2	3	4
32	$\bar{1}$·9284	9279	9275	9270	9265	9260	9255	9251	9246	9241	1	2	2	3	4
33	$\bar{1}$·9236	9231	9226	9221	9216	9211	9206	9201	9196	9191	1	2	3	3	4
34	$\bar{1}$·9186	9181	9175	9170	9165	9160	9155	9149	9144	9139	1	2	3	3	4
35	$\bar{1}$·9134	9128	9123	9118	9112	9107	9101	9096	9091	9085	1	2	3	4	5
36	$\bar{1}$·9080	9074	9069	9063	9057	9052	9046	9041	9035	9029	1	2	3	4	5
37	$\bar{1}$·9023	9018	9012	9006	9000	8995	8989	8983	8977	8971	1	2	3	4	5
38	$\bar{1}$·8965	8959	8953	8947	8941	8935	8929	8923	8917	8911	1	2	3	4	5
39	$\bar{1}$·8905	8899	8893	8887	8880	8874	8868	8862	8855	8849	1	2	3	4	5
40	$\bar{1}$·8843	8836	8830	8823	8817	8810	8804	8797	8791	8784	1	2	3	4	5
41	$\bar{1}$·8778	8771	8765	8758	8751	8745	8738	8731	8724	8718	1	2	3	5	6
42	$\bar{1}$·8711	8704	8697	8690	8683	8676	8669	8662	8655	8648	1	2	3	5	6
43	$\bar{1}$·8641	8634	8627	8620	8613	8606	8598	8591	8584	8577	1	2	4	5	6
44	$\bar{1}$·8569	8562	8555	8547	8540	8532	8525	8517	8510	8502	1	2	4	5	6

LOGARITHMS OF COSINES

[Numbers in difference columns to be subtracted, not added.]

Degrees	0′ 0°·0	6′ 0°·1	12′ 0°·2	18′ 0°·3	24′ 0°·4	30′ 0°·5	36′ 0°·6	42′ 0°·7	48′ 0°·8	54′ 0°·9	Mean Differences 1 2 3	4 5
45	$\bar{1}$·8495	8487	8480	8472	8464	8457	8449	8441	8433	8426	1 3 4	5 6
46	$\bar{1}$·8418	8410	8402	8394	8386	8378	8370	8362	8354	8346	1 3 4	5 7
47	$\bar{1}$·8338	8330	8322	8313	8305	8297	8289	8280	8272	8264	1 3 4	6 7
48	$\bar{1}$·8255	8247	8238	8230	8221	8213	8204	8195	8187	8178	1 3 4	6 7
49	$\bar{1}$·8169	8161	8152	8143	8134	8125	8117	8108	8099	8090	1 3 4	6 7
50	$\bar{1}$·8081	8072	8063	8053	8044	8035	8026	8017	8007	7998	2 3 5	6 8
51	$\bar{1}$·7989	7979	7970	7960	7951	7941	7932	7922	7913	7903	2 3 5	6 8
52	$\bar{1}$·7893	7884	7874	7864	7854	7844	7835	7825	7815	7805	2 3 5	7 8
53	$\bar{1}$·7795	7785	7774	7764	7754	7744	7734	7723	7713	7703	2 3 5	7 9
54	$\bar{1}$·7692	7682	7671	7661	7650	7640	7629	7618	7607	7597	2 4 5	7 9
55	$\bar{1}$·7586	7575	7564	7553	7542	7531	7520	7509	7498	7487	2 4 6	7 9
56	$\bar{1}$·7476	7464	7453	7442	7430	7419	7407	7396	7384	7373	2 4 6	8 10
57	$\bar{1}$·7361	7349	7338	7326	7314	7302	7290	7278	7266	7254	2 4 6	8 10
58	$\bar{1}$·7242	7230	7218	7205	7193	7181	7168	7156	7144	7131	2 4 6	8 10
59	$\bar{1}$·7118	7106	7093	7080	7068	7055	7042	7029	7016	7003	2 4 6	9 11
60	$\bar{1}$·6990	6977	6963	6950	6937	6923	6910	6896	6883	6869	2 4 7	9 11
61	$\bar{1}$·6856	6842	6828	6814	6801	6787	6773	6759	6744	6730	2 5 7	9 12
62	$\bar{1}$·6716	6702	6687	6673	6659	6644	6629	6615	6600	6585	2 5 7	10 12
63	$\bar{1}$·6570	6556	6541	6526	6510	6495	6480	6465	6449	6434	3 5 8	10 13
64	$\bar{1}$·6418	6403	6387	6371	6356	6340	6324	6308	6292	6276	3 5 8	11 13
65	$\bar{1}$·6259	6243	6227	6210	6194	6177	6161	6144	6127	6110	3 6 8	11 14
66	$\bar{1}$·6093	6076	6059	6042	6024	6007	5990	5972	5954	5937	3 6 9	12 15
67	$\bar{1}$·5919	5901	5883	5865	5847	5828	5810	5792	5773	5754	3 6 9	12 15
68	$\bar{1}$·5736	5717	5698	5679	5660	5641	5621	5602	5583	5563	3 6 10	13 16
69	$\bar{1}$·5543	5523	5504	5484	5463	5443	5423	5402	5382	5361	3 7 10	14 17
70	$\bar{1}$·5341	5320	5299	5278	5256	5235	5213	5192	5170	5148	4 7 11	14 18
71	$\bar{1}$·5126	5104	5082	5060	5037	5015	4992	4969	4946	4923	4 8 11	15 19
72	$\bar{1}$·4900	4876	4853	4829	4805	4781	4757	4733	4709	4684	4 8 12	16 20
73	$\bar{1}$·4659	4634	4609	4584	4559	4533	4508	4482	4456	4430	4 9 13	17 21
74	$\bar{1}$·4403	4377	4350	4323	4296	4269	4242	4214	4186	4158	5 9 14	18 23
75	$\bar{1}$·4130	4102	4073	4044	4015	3986	3957	3927	3897	3867	5 10 15	20 24
76	$\bar{1}$·3837	3806	3775	3745	3713	3682	3650	3618	3586	3554	5 11 16	21 26
77	$\bar{1}$·3521	3488	3455	3421	3387	3353	3319	3284	3250	3214	6 11 17	23 28
78	$\bar{1}$·3179	3143	3107	3070	3034	2997	2959	2921	2883	2845	6 12 19	25 31
79	$\bar{1}$·2806	2767	2727	2687	2647	2606	2565	2524	2482	2439	7 14 20	27 34
80	$\bar{1}$·2397	2353	2310	2266	2221	2176	2131	2085	2038	1991	8 15 23	30 38
81	$\bar{1}$·1943	1895	1847	1797	1747	1697	1646	1594	1542	1489	8 17 25	34 42
82	$\bar{1}$·1430	1381	1326	1271	1214	1157	1099	1040	0981	0920	10 19 29	38 48
83	$\bar{1}$·0859	0797	0734	0670	0605	0539	0472	0403	0334	0264	11 22 33	44 55
84	$\bar{1}$·0192	0120	0046	$\bar{2}$·9970	$\bar{2}$·9894	$\bar{2}$·9816	$\bar{2}$·9736	$\bar{2}$·9655	$\bar{2}$·9573	$\bar{2}$·9489	13 26 39	52 65
85	$\bar{2}$·9403	9315	9226	9135	9042	8946	8849	8749	8647	8543	16 32 48	64 80
86	$\bar{2}$·8436	8326	8213	8098	7979	7857	7731	7602	7468	7330		
87	$\bar{2}$·7188	7041	6889	6731	6567	6397	6220	6035	5842	5640		
88	$\bar{2}$·5428	5206	4971	4723	4459	4179	3880	3558	3210	2832		
89	$\bar{2}$·2419	1961	1450	0870	0200	$\bar{3}$·9408	$\bar{3}$·8439	$\bar{3}$·7190	$\bar{3}$·5429	$\bar{3}$·2419		
90	∞											

Degrees	0′ 0°·0	6′ 0°·1	12′ 0°·2	18′ 0°·3	24′ 0°·4	30′ 0°·5	36′ 0°·6	42′ 0°·7	48′ 0°·8	54′ 0°·9	Mean Differences 1 2 3	Mean Differences 4 5
0	−∞	3·2419	3·5429	3·7190	3·8439	3·9409	2·0200	2·0870	2·1450	2·1962		
1	2·2419	2833	3211	3559	3881	4181	4461	4725	4973	5208		
2	2·5431	5643	5845	6038	6223	6401	6571	6736	6894	7046		
3	2·7194	7337	7475	7609	7739	7865	7988	8107	8223	8336		
4	2·8446	8554	8659	8762	8862	8960	9056	9150	9241	9331	16 32 48	64 81
5	2·9420	9506	9591	9674	9756	9836	9915	9992	1·0068	1·0143	13 26 40	53 66
6	1·0216	0289	0360	0430	0499	0567	0633	0699	0764	0828	11 22 34	45 56
7	1·0891	0954	1015	1076	1135	1194	1252	1310	1367	1423	10 20 29	39 49
8	1·1478	1533	1587	1640	1693	1745	1797	1848	1898	1948	9 17 26	35 43
9	1·1997	2046	2094	2142	2189	2236	2282	2328	2374	2419	8 16 23	31 39
10	1·2463	2507	2551	2594	2637	2680	2722	2764	2805	2846	7 14 21	28 35
11	1·2887	2927	2967	3006	3046	3085	3123	3102	3200	3237	6 13 19	26 32
12	1·3275	3312	3349	3385	3422	3458	3493	3529	3564	3599	6 12 18	24 30
13	1·3634	3668	3702	3736	3770	3804	3837	3870	3903	3935	6 11 17	22 28
14	1·3968	4000	4032	4064	4095	4127	4158	4189	4220	4250	5 10 16	21 26
15	1·4281	4311	4341	4371	4400	4430	4459	4488	4517	4546	5 10 15	20 25
16	1·4575	4603	4632	4660	4688	4716	4744	4771	4799	4826	5 9 14	19 23
17	1·4853	4880	4907	4934	4961	4987	5014	5040	5066	5092	4 9 13	18 22
18	1·5118	5143	5169	5195	5220	5245	5270	5295	5320	5345	4 8 13	17 21
19	1·5370	5394	5419	5443	5467	5491	5516	5539	5563	5587	4 8 12	16 20
20	1·5611	5634	5658	5681	5704	5727	5750	5773	5796	5819	4 8 12	15 19
21	1·5842	5864	5887	5909	5932	5954	5976	5998	6020	6042	4 7 11	15 19
22	1·6064	6086	6108	6129	6151	6172	6194	6215	6236	6257	4 7 11	14 18
23	1·6279	6300	6321	6341	6362	6383	6404	6424	6445	6465	3 7 10	14 17
24	1·6486	6506	6527	6547	6567	6587	6607	6627	6647	6667	3 7 10	13 17
25	1·6687	6706	6726	6746	6765	6785	6804	6824	6843	6863	3 7 10	13 16
26	1·6882	6901	6920	6939	6958	6977	6996	7015	7034	7053	3 6 9	13 16
27	1·7072	7090	7109	7128	7146	7165	7183	7202	7220	7238	3 6 9	12 15
28	1·7257	7275	7293	7311	7330	7348	7366	7384	7402	7420	3 6 9	12 15
29	1·7438	7455	7473	7491	7509	7526	7544	7562	7579	7597	3 6 9	12 15
30	1·7614	7632	7649	7667	7684	7701	7719	7736	7753	7771	3 6 9	12 14
31	1·7788	7805	7822	7839	7856	7873	7890	7907	7924	7941	3 6 9	11 14
32	1·7958	7975	7992	8008	8025	8042	8059	8075	8092	8109	3 6 8	11 14
33	1·8125	8142	8158	8175	8191	8208	8224	8241	8257	8274	3 5 8	11 14
34	1·8290	8306	8323	8339	8355	8371	8388	8404	8420	8436	3 5 8	11 14
35	1·8452	8468	8484	8501	8517	8533	8549	8565	8581	8597	3 5 8	11 13
36	1·8613	8629	8644	8660	8676	8692	8708	8724	8740	8755	3 5 8	11 13
37	1·8771	8787	8803	8818	8834	8850	8865	8881	8897	8912	3 5 8	10 13
38	1·8928	8944	8959	8975	8990	9006	9022	9037	9053	9068	3 5 8	10 13
39	1·9084	9099	9115	9130	9146	9161	9176	9192	9207	9223	3 5 8	10 13
40	1·9238	9254	9269	9284	9300	9315	9330	9346	9361	9376	3 5 8	10 13
41	1·9392	9407	9422	9438	9453	9468	9483	9499	9514	9529	3 5 8	10 13
42	1·9544	9560	9575	9590	9605	9621	9636	9651	9666	9681	3 5 8	10 13
43	1·9697	9712	9727	9742	9757	9772	9788	9803	9818	9833	3 5 8	10 13
44	1·9848	9864	9879	9894	9909	9924	9939	9955	9970	9985	3 5 8	10 13

Degrees	0′ 0°·0	6′ 0°·1	12′ 0°·2	18′ 0°·3	24′ 0°·4	30′ 0°·5	36′ 0°·6	42′ 0°·7	48′ 0°·8	54′ 0°·9	Mean Differences 1	2	3	4	5
45	·0000	0015	0030	0045	0061	0076	0091	0106	0121	0136	3	5	8	10	13
46	·0152	0167	0182	0197	0212	0228	0243	0258	0273	0288	3	5	8	10	13
47	·0303	0319	0334	0349	0364	0379	0395	0410	0425	0440	3	5	8	10	13
48	·0456	0471	0486	0501	0517	0532	0547	0562	0578	0593	3	5	8	10	13
49	·0608	0624	0639	0654	0670	0685	0700	0716	0731	0746	3	5	8	10	13
50	·0762	0777	0793	0808	0824	0839	0854	0870	0885	0901	3	5	8	10	13
51	·0916	0932	0947	0963	0978	0994	1010	1025	1041	1056	3	5	8	10	13
52	·1072	1088	1103	1119	1135	1150	1166	1182	1197	1213	3	5	8	10	13
53	·1229	1245	1260	1276	1292	1308	1324	1340	1356	1371	3	5	8	11	13
54	·1387	1403	1419	1435	1451	1467	1483	1499	1516	1532	3	5	8	11	13
55	·1548	1564	1580	1596	1612	1629	1645	1661	1677	1694	3	5	8	11	14
56	·1710	1726	1743	1759	1776	1792	1809	1825	1842	1858	3	5	8	11	14
57	·1875	1891	1908	1925	1941	1958	1975	1992	2008	2025	3	6	8	11	14
58	·2042	2059	2076	2093	2110	2127	2144	2161	2178	2195	3	6	9	11	14
59	·2212	2229	2247	2264	2281	2299	2316	2333	2351	2368	3	6	9	12	14
60	·2386	2403	2421	2438	2456	2474	2491	2509	2527	2545	3	6	9	12	15
61	·2562	2580	2598	2616	2634	2652	2670	2689	2707	2725	3	6	9	12	15
62	·2743	2762	2780	2798	2817	2835	2854	2872	2891	2910	3	6	9	12	15
63	·2928	2947	2966	2985	3004	3023	3042	3061	3080	3099	3	6	9	13	16
64	·3118	3137	3157	3176	3196	3215	3235	3254	3274	3294	3	6	10	13	16
65	·3313	3333	3353	3373	3393	3413	3433	3453	3473	3494	3	7	10	13	17
66	·3514	3535	3555	3576	3596	3617	3638	3659	3679	3700	3	7	10	14	17
67	·3721	3743	3764	3785	3806	3828	3849	3871	3892	3914	4	7	11	14	18
68	·3936	3958	3980	4002	4024	4046	4068	4091	4113	4136	4	7	11	15	19
69	·4158	4181	4204	4227	4250	4273	4296	4319	4342	4366	4	8	12	15	19
70	·4389	4413	4437	4461	4484	4509	4533	4557	4581	4606	4	8	12	16	20
71	·4630	4655	4680	4705	4730	4755	4780	4805	4831	4857	4	8	13	17	21
72	·4882	4908	4934	4960	4986	5013	5039	5066	5093	5120	4	9	13	18	22
73	·5147	5174	5201	5229	5256	5284	5312	5340	5368	5397	5	9	14	19	23
74	·5425	5454	5483	5512	5541	5570	5600	5629	5659	5689	5	10	15	20	25
75	·5719	5750	5780	5811	5842	5873	5905	5936	5968	6000	5	10	16	21	26
76	·6032	6065	6097	6130	6163	6196	6230	6264	6298	6332	6	11	17	22	28
77	·6366	6401	6436	6471	6507	6542	6578	6615	6651	6688	6	12	18	24	30
78	·6725	6763	6800	6838	6877	6915	6954	6994	7033	7073	6	13	19	26	32
79	·7113	7154	7195	7236	7278	7320	7363	7406	7449	7493	7	14	21	28	35
80	·7537	7581	7626	7672	7718	7764	7811	7858	7906	7954	8	16	23	31	39
81	·8003	8052	8102	8152	8203	8255	8307	8360	8413	8467	9	17	26	35	43
82	·8522	8577	8633	8690	8748	8806	8865	8924	8985	9046	10	20	29	39	49
83	·9109	9172	9236	9301	9367	9433	9501	9570	9640	9711	11	22	34	45	56
84	·9784	9857	9932	1·0008	1·0085	1·0164	1·0244	1·0326	1·0409	1·0494	13	26	40	53	66
85	1·0580	0669	0759	0850	0944	1040	1138	1238	1341	1446	16	32	48	64	81
86	1·1554	1664	1777	1893	2012	2135	2261	2391	2525	2663					
87	1·2806	2954	3106	3264	3429	3599	3777	3962	4155	4357					
88	1·4569	4792	5027	5275	5539	5819	6119	6441	6789	7167					
89	1·7581	8038	8550	9130	9800	2·0591	2·1561	2·2810	2·4571	2·7581					

HYPERBOLIC OR NAPERIAN LOGARITHMS

	0	1	2	3	4	5	6	7	8	9	Mean Differences 1 2 3	4 5 6	7 8 9
1·0	0·0000	0099	0198	0296	0392	0488	0583	0677	0770	0862	10 19 29	38 48 57	67 76 86
1·1	·0953	1044	1133	1222	1310	1398	1484	1570	1655	1740	9 17 26	35 44 52	61 70 78
1·2	·1823	1906	1989	2070	2151	2231	2311	2390	2469	2546	8 16 24	32 40 48	56 64 72
1·3	·2624	2700	2776	2852	2927	3001	3075	3148	3221	3293	7 15 22	30 37 44	52 59 67
1·4	·3365	3436	3507	3577	3646	3716	3784	3853	3920	3988	7 14 21	28 35 41	48 55 62
1·5	·4055	4121	4187	4253	4318	4383	4447	4511	4574	4637	6 13 19	26 32 39	45 52 58
1·6	·4700	4762	4824	4886	4947	5008	5068	5128	5188	5247	6 12 18	24 30 36	42 48 55
1·7	·5306	5365	5423	5481	5539	5596	5653	5710	5766	5822	6 11 17	24 29 34	40 46 51
1·8	·5878	5933	5988	6043	6098	6152	6206	6259	6313	6366	5 11 16	22 27 32	38 43 49
1·9	·6419	6471	6523	6575	6627	6678	6729	6780	6831	6881	5 10 15	20 26 31	36 41 46
2·0	·6931	6981	7031	7080	7129	7178	7227	7275	7324	7372	5 10 15	20 24 29	34 39 44
2·1	·7419	7467	7514	7561	7608	7655	7701	7747	7793	7839	5 9 14	19 23 28	33 37 42
2·2	·7885	7930	7975	8020	8065	8109	8154	8198	8242	8286	4 9 13	18 22 27	31 36 40
2·3	·8329	8372	8416	8459	8502	8544	8587	8629	8671	8713	4 9 13	17 21 26	30 34 38
2·4	·8755	8796	8838	8879	8920	8961	9002	9042	9083	9123	4 8 12	16 20 24	29 33 37
2·5	·9163	9203	9243	9282	9322	9361	9400	9439	9478	9517	4 8 12	16 20 24	27 31 35
2·6	·9555	9594	9632	9670	9708	9746	9783	9821	9858	9895	4 8 11	15 19 23	26 30 34
2·7	·9933	9969	1·0006	0043	0080	0116	0152	0188	0225	0260	4 7 11	15 18 22	25 29 33
2·8	1·0296	0332	0367	0403	0438	0473	0508	0543	0578	0613	4 7 11	14 18 21	25 28 32
2·9	1·0647	0682	0716	0750	0784	0818	0852	0886	0919	0953	3 7 10	14 17 20	24 27 31
3·0	1·0986	1019	1053	1086	1119	1151	1184	1217	1249	1282	3 7 10	13 16 20	23 26 30
3·1	1·1314	1346	1378	1410	1442	1474	1506	1537	1569	1600	3 6 10	13 16 19	22 25 29
3·2	1·1632	1663	1694	1725	1756	1787	1817	1848	1878	1909	3 6 9	12 15 18	22 25 28
3·3	1·1939	1969	1·2000	2030	2060	2090	2119	2149	2179	2208	3 6 9	12 15 18	21 24 27
3·4	1·2238	2267	2296	2326	2355	2384	2413	2442	2470	2499	3 6 9	12 15 17	20 23 26
3·5	1·2528	2556	2585	2613	2641	2669	2698	2726	2754	2782	3 6 8	11 14 17	20 23 25
3·6	1·2809	2837	2865	2892	2920	2947	2975	3002	3029	3056	3 5 8	11 14 16	19 22 25
3·7	1·3083	3110	3137	3164	3191	3218	3244	3271	3297	3324	3 5 8	11 13 16	19 21 24
3·8	1·3350	3376	3403	3429	3455	3481	3507	3533	3558	3584	3 5 8	10 13 16	18 21 23
3·9	1·3610	3635	3661	3686	3712	3737	3762	3788	3813	3838	3 5 8	10 13 15	18 20 23
4·0	1·3863	3888	3913	3938	3962	3987	4012	4036	4061	4085	2 5 7	10 12 15	17 20 22
4·1	1·4110	4134	4159	4183	4207	4231	4255	4279	4303	4327	2 5 7	10 12 14	17 19 22
4·2	1·4351	4375	4398	4422	4446	4469	4493	4516	4540	4563	2 5 7	9 12 14	16 19 21
4·3	1·4586	4609	4633	4656	4679	4702	4725	4748	4770	4793	2 5 7	9 12 14	16 18 21
4·4	1·4816	4839	4861	4884	4907	4929	4951	4974	4996	5019	2 5 7	9 11 14	16 18 20
4·5	1·5041	5063	5085	5107	5129	5151	5173	5195	5217	5239	2 4 7	9 11 13	15 18 20
4·6	1·5261	5282	5304	5326	5347	5369	5390	5412	5433	5454	2 4 6	9 11 13	15 17 19
4·7	1·5476	5497	5518	5539	5560	5581	5602	5623	5644	5665	2 4 6	8 11 13	15 17 19
4·8	1·5686	5707	5728	5748	5769	5790	5810	5831	5851	5872	2 4 6	8 10 12	14 16 19
4·9	1·5892	5913	5933	5953	5974	5994	6014	6034	6054	6074	2 4 6	8 10 12	14 16 18
5·0	1·6094	6114	6134	6154	6174	6194	6214	6233	6253	6273	2 4 6	8 10 12	14 16 18
5·1	1·6292	6312	6332	6351	6371	6390	6409	6429	6448	6467	2 4 6	8 10 12	14 16 18
5·2	1·6487	6506	6525	6544	6563	6582	6601	6620	6639	6658	2 4 6	8 10 11	13 15 17
5·3	1·6677	6696	6715	6734	6752	6771	6790	6808	6827	6845	2 4 6	7 9 11	13 15 17
5·4	1·6864	6882	6901	6919	6938	6956	6974	6993	7011	7029	2 4 5	7 9 11	13 15 17

Hyperbolic or Naperian Logarithms of 10^{+n}.

n	1	2	3	4	5	6	7	8	9
$\log_e 10^n$	2·3026	4·6052	6·9078	9·2103	11·5129	13·8155	16·1181	18·4207	20·7233

	0	1	2	3	4	5	6	7	8	9	Mean Differences					
											1 2 3		4 5 6		7 8	
5·5	1·7047	7066	7084	7102	7120	7138	7156	7174	7192	7210	2 4 5		7 9 11		13 14	
5·6	1·7228	7246	7263	7281	7299	7317	7334	7352	7370	7387	2 4 5		7 9 11		12 14	
5·7	1·7405	7422	7440	7457	7475	7492	7509	7527	7544	7561	2 3 5		7 9 10		12 14	
5·8	1·7579	7596	7613	7630	7647	7664	7681	7699	7716	7733	2 3 5		7 9 10		12 14	
5·9	1·7750	7766	7783	7800	7817	7834	7851	7867	7884	7901	2 3 5		7 8 10		12 13	
6·0	1·7918	7934	7951	7967	7984	8001	8017	8034	8050	8066	2 3 5		7 8 10		12 13	
6·1	1·8083	8099	8116	8132	8148	8165	8181	8197	8213	8229	2 3 5		6 8 10		11 13	
6·2	1·8245	8262	8278	8294	8310	8326	8342	8358	8374	8390	2 3 5		6 8 10		11 13	
6·3	1·8405	8421	8437	8453	8469	8485	8500	8516	8532	8547	2 3 5		6 8 9		11 13	
6·4	1·8563	8579	8594	8610	8625	8641	8656	8672	8687	8703	2 3 5		6 8 9		11 12	
6·5	1·8718	8733	8749	8764	8779	8795	8810	8825	8840	8856	2 3 5		6 8 9		11 12	
6·6	1·8871	8886	8901	8916	8931	8946	8961	8976	8991	9006	2 3 5		6 8 9		11 12	
6·7	1·9021	9036	9051	9066	9081	9095	9110	9125	9140	9155	1 3 4		6 7 9		10 12	
6·8	1·9169	9184	9199	9213	9228	9242	9257	9272	9286	9301	1 3 4		6 7 9		10 12	
6·9	1·9315	9330	9344	9359	9373	9387	9402	9416	9430	9445	1 3 4		6 7 9		10 12	
7·0	1·9459	9473	9488	9502	9516	9530	9544	9559	9573	9587	1 3 4		6 7 9		10 11	
7·1	1·9601	9615	9629	9643	9657	9671	9685	9699	9713	9727	1 3 4		6 7 8		10 11	
7·2	1·9741	9755	9769	9782	9796	9810	9824	9838	9851	9865	1 3 4		6 7 8		10 11	
7·3	1·9879	9892	9906	9920	9933	9947	9961	9974	9988	2·0001	1 3 4		5 7 8		10 11	
7·4	2·0015	0028	0042	0055	0069	0082	0096	0109	0122	0136	1 3 4		5 7 8		9 11	
7·5	2·0149	0162	0176	0189	0202	0215	0229	0242	0255	0268	1 3 4		5 7 8		9 11	
7·6	2·0281	0295	0308	0321	0334	0347	0360	0373	0386	0399	1 3 4		5 7 8		9 10	
7·7	2·0412	0425	0438	0451	0464	0477	0490	0503	0516	0528	1 3 4		5 6 8		9 10	
7·8	2·0541	0554	0567	0580	0592	0605	0618	0631	0643	0656	1 3 4		5 6 8		9 10	
7·9	2·0669	0681	0694	0707	0719	0732	0744	0757	0769	0782	1 3 4		5 6 8		9 10	
8·0	2·0794	0807	0819	0832	0844	0857	0869	0882	0894	0906	1 3 4		5 6 7		9 10	
8·1	2·0919	0931	0943	0956	0968	0980	0992	1005	1017	1029	1 2 4		5 6 7		9 10	
8·2	2·1041	1054	1066	1078	1090	1102	1114	1126	1138	1150	1 2 4		5 6 7		9 10	
8·3	2·1163	1175	1187	1199	1211	1223	1235	1247	1258	1270	1 2 4		5 6 7		8 10	
8·4	2·1282	1294	1306	1318	1330	1342	1353	1365	1377	1389	1 2 4		5 6 7		8 9	
8·5	2·1401	1412	1424	1436	1448	1459	1471	1483	1494	1506	1 2 4		5 6 7		8 9	
8·6	2·1518	1529	1541	1552	1564	1576	1587	1599	1610	1622	1 2 3		5 6 7		8 9	
8·7	2·1633	1645	1656	1668	1679	1691	1702	1713	1725	1736	1 2 3		5 6 7		8 9	
8·8	2·1748	1759	1770	1782	1793	1804	1815	1827	1838	1849	1 2 3		5 6 7		8 9	
8·9	2·1861	1872	1883	1894	1905	1917	1928	1939	1950	1961	1 2 3		4 6 7		8 9	
9·0	2·1972	1983	1994	2006	2017	2028	2039	2050	2061	2072	1 2 3		4 6 7		8 9	
9·1	2·2083	2094	2105	2116	2127	2138	2148	2159	2170	2181	1 2 3		4 5 7		8 9	
9·2	2·2192	2203	2214	2225	2235	2246	2257	2268	2279	2289	1 2 3		4 5 6		8 9	
9·3	2·2300	2311	2322	2332	2343	2354	2364	2375	2386	2396	1 2 3		4 5 6		7 9	
9·4	2·2407	2418	2428	2439	2450	2460	2471	2481	2492	2502	1 2 3		4 5 6		7 8	
9·5	2·2513	2523	2534	2544	2555	2565	2576	2586	2597	2607	1 2 3		4 5 6		7 8	
9·6	2·2618	2628	2638	2649	2659	2670	2680	2690	2701	2711	1 2 3		4 5 6		7 8	
9·7	2·2721	2732	2742	2752	2762	2773	2783	2793	2803	2814	1 2 3		4 5 6		7 8	
9·8	2·2824	2834	2844	2854	2865	2875	2885	2895	2905	2915	1 2 3		4 5 6		7 8	
9·9	2·2925	2935	2946	2956	2966	2976	2986	2996	3006	3016	1 2 3		4 5 6		7 8	
10·0	2·3026															

Hyperbolic or Naperian Logarithms of 10^{-n}.

n	1	2	3	4	5	6	7	8	9
$\log_e 10^{-n}$	$\bar{3}\cdot6974$	$\bar{5}\cdot3948$	$\bar{7}\cdot0922$	$\bar{10}\cdot7897$	$\bar{12}\cdot4871$	$\bar{14}\cdot1845$	$\bar{17}\cdot8819$	$\bar{19}\cdot5793$	$\bar{21}\cdot2767$

POWERS, ROOTS AND RECIPROCALS

n	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$	$\sqrt{10n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$	$\frac{1}{n}$
1	1	1	1	1	3·162	2·154	4·642	1
2	4	8	1·414	1·260	4·472	2·714	5·848	·5000
3	9	27	1·732	1·442	5·477	3·107	6·694	·3333
4	16	64	2	1·587	6·325	3·420	7·368	·2500
5	25	125	2·236	1·710	7·071	3·684	7·937	·2000
6	36	216	2·449	1·817	7·746	3·915	8·434	·1667
7	49	343	2·646	1·913	8·367	4·121	8·879	·1429
8	64	512	2·828	2·000	8·944	4·309	9·283	·1250
9	81	729	3·000	2·080	9·487	4·481	9·655	·1111
10	100	1000	3·162	2·154	10·0	4·642	10·000	·1000
11	121	1331	3·317	2·224	10·488	4·791	10·323	·09091
12	144	1728	3·464	2·289	10·954	4·932	10·627	·08333
13	169	2197	3·606	2·351	11·402	5·066	10·914	·07692
14	196	2744	3·742	2·410	11·832	5·192	11·187	·07143
15	225	3375	3·873	2·466	12·247	5·313	11·447	·06667
16	256	4096	4·000	2·520	12·649	5·429	11·696	·06250
17	289	4913	4·123	2·571	13·038	5·540	11·935	·05882
18	324	5832	4·243	2·621	13·416	5·646	12·164	·05556
19	361	6859	4·359	2·668	13·784	5·749	12·386	·05263
20	400	8000	4·472	2·714	14·142	5·848	12·599	·0500
21	441	9261	4·583	2·759	14·491	5·944	12·806	·04762
22	484	10648	4·690	2·802	14·832	6·037	13·006	·04545
23	529	12167	4·796	2·844	15·166	6·127	13·200	·04348
24	576	13824	4·899	2·884	15·492	6·214	13·389	·04167
25	625	15625	5·000	2·924	15·811	6·300	13·572	·0400
26	676	17576	5·099	2·962	16·125	6·383	13·751	·03846
27	729	19683	5·196	3·000	16·432	6·463	13·925	·03704
28	784	21952	5·292	3·037	16·733	6·542	14·095	·03571
29	841	24389	5·385	3·072	17·029	6·619	14·260	·03448
30	900	27000	5·477	3·107	17·321	6·694	14·422	·03333
31	961	29791	5·568	3·141	17·607	6·768	14·581	·03226
32	1024	32768	5·657	3·175	17·889	6·840	14·736	·03125
33	1089	35937	5·745	3·208	18·166	6·910	14·888	·03030
34	1156	39304	5·831	3·240	18·439	6·980	15·037	·02941
35	1225	42875	5·916	3·271	18·708	7·047	15·183	·02857
36	1296	46656	6·000	3·302	18·974	7·114	15·326	·02778
37	1369	50653	6·083	3·332	19·235	7·179	15·467	·02703
38	1444	54872	6·164	3·362	19·494	7·243	15·605	·02632
39	1521	59319	6·245	3·391	19·748	7·306	15·741	·02564
40	1600	64000	6·325	3·420	20·00	7·368	15·874	·0250
41	1681	68921	6·403	3·448	20·248	7·429	16·005	·02439
42	1764	74088	6·481	3·476	20·494	7·489	16·134	·02381
43	1849	79507	6·557	3·503	20·736	7·548	16·261	·02326
44	1936	85184	6·633	3·530	20·976	7·606	16·386	·02273
45	2025	91125	6·708	3·557	21·213	7·663	16·510	·02222
46	2116	97336	6·782	3·583	21·448	7·719	16·631	·02174
47	2209	103823	6·856	3·609	21·679	7·775	16·751	·02128
48	2304	110592	6·928	3·634	21·909	7·830	16·869	·02083
49	2401	117649	7·000	3·659	22·136	7·884	16·985	·02041
50	2500	125000	7·071	3·684	22·361	7·937	17·100	·020

n	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$	$\sqrt{10n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$	$\frac{1}{n}$
51	2601	132651	7·141	3·708	22·583	7·990	17·213	·01961
52	2704	140608	7·211	3·733	22·804	8·041	17·325	·01923
53	2809	148877	7·280	3·756	23·022	8·093	17·435	·01887
54	2916	157464	7·348	3·780	23·238	8·143	17·544	·01852
55	3025	166375	7·416	3·803	23·452	8·193	17·652	·01818
56	3136	175616	7·483	3·826	23·664	8·243	17·758	·01786
57	3249	185193	7·550	3·849	23·875	8·291	17·863	·01754
58	3364	195112	7·616	3·871	24·083	8·340	17·967	·01724
59	3481	205379	7·681	3·893	24·290	8·387	18·070	·01695
60	3600	216000	7·746	3·915	24·495	8·434	18·171	·01667
61	3721	226981	7·810	3·936	24·698	8·481	18·272	·01639
62	3844	238328	7·874	3·958	24·900	8·527	18·371	·01613
63	3969	250047	7·937	3·979	25·100	8·573	18·469	·01587
64	4096	262144	8·000	4·000	25·298	8·618	18·566	·01562
65	4225	274625	8·062	4·021	25·495	8·662	18·663	·01538
66	4356	287496	8·124	4·041	25·690	8·707	18·758	·01515
67	4489	300763	8·185	4·062	25·884	8·750	18·852	·01493
68	4624	314432	8·246	4·082	26·077	8·794	18·945	·01471
69	4761	328509	8·307	4·102	26·268	8·837	19·038	·01449
70	4900	343000	8·367	4·121	26·458	8·879	19·129	·01429
71	5041	357911	8·426	4·141	26·646	8·921	19·220	·01408
72	5184	373248	8·485	4·160	26·833	8·963	19·310	·01389
73	5329	389017	8·544	4·179	27·019	9·004	19·399	·01370
74	5476	405224	8·602	4·198	27·203	9·045	19·487	·01351
75	5625	421875	8·660	4·217	27·386	9·086	19·574	·01333
76	5776	438976	8·718	4·236	27·568	9·126	19·661	·01316
77	5929	456533	8·775	4·254	27·749	9·166	19·747	·01299
78	6084	474552	8·832	4·273	27·928	9·205	19·832	·01282
79	6241	493039	8·888	4·291	28·107	9·244	19·916	·01266
80	6400	512000	8·944	4·309	28·284	9·283	20·000	·01250
81	6561	531441	9·000	4·327	28·460	9·322	20·083	·01235
82	6724	551368	9·055	4·344	28·636	9·360	20·165	·01220
83	6889	571787	9·110	4·362	28·810	9·398	20·247	·01205
84	7056	592704	9·165	4·380	28·983	9·435	20·328	·01190
85	7225	614125	9·220	4·397	29·155	9·473	20·408	·01176
86	7396	636056	9·274	4·414	29·326	9·510	20·488	·01163
87	7569	658503	9·327	4·431	29·496	9·546	20·567	·01149
88	7744	681472	9·381	4·448	29·665	9·583	20·646	·01136
89	7921	704969	9·434	4·465	29·833	9·619	20·724	·01124
90	8100	729000	9·487	4·481	30·000	9·655	20·801	·01111
91	8281	753571	9·539	4·498	30·166	9·691	20·878	·01099
92	8464	778688	9·592	4·514	30·332	9·726	20·954	·01087
93	8649	804357	9·644	4·531	30·496	9·761	21·029	·01075
94	8836	830584	9·695	4·547	30·659	9·796	21·105	·01064
95	9025	857375	9·747	4·563	30·822	9·830	21·179	·01053
96	9216	884736	9·798	4·579	30·984	9·865	21·253	·01042
97	9409	912673	9·849	4·595	31·145	9·899	21·327	·01031
98	9604	941192	9·899	4·610	31·305	9·933	21·400	·01020
99	9801	970299	9·950	4·626	31·464	9·967	21·472	·01010
100	10000	1000000	10·000	4·642	31·623	10·000	21·544	·0100

SQUARE ROOTS. From 1 to 10

	0	1	2	3	4	5	6	7	8	9	Mean Differences		
											1 2 3	4 5 6	7 8 9
1·0	1·000	1·005	1·010	1·015	1·020	1·025	1·030	1·034	1·039	1·044	0 1 1	2 2 3	3 4 4
1·1	1·049	1·054	1·058	1·063	1·068	1·072	1·077	1·082	1·086	1·091	0 1 1	2 2 3	3 4 4
1·2	1·095	1·100	1·105	1·109	1·114	1·118	1·122	1·127	1·131	1·136	0 1 1	2 2 3	3 4 4
1·3	1·140	1·145	1·149	1·153	1·158	1·162	1·166	1·170	1·175	1·179	0 1 1	2 2 3	3 3 4
1·4	1·183	1·187	1·192	1·196	1·200	1·204	1·208	1·212	1·217	1·221	0 1 1	2 2 2	3 3 4
1·5	1·225	1·229	1·233	1·237	1·241	1·245	1·249	1·253	1·257	1·261	0 1 1	2 2 2	3 3 4
1·6	1·265	1·269	1·273	1·277	1·281	1·285	1·288	1·292	1·296	1·300	0 1 1	2 2 2	3 3 3
1·7	1·304	1·308	1·311	1·315	1·319	1·323	1·327	1·330	1·334	1·338	0 1 1	2 2 2	3 3 3
1·8	1·342	1·345	1·349	1·353	1·350	1·360	1·364	1·367	1·371	1·375	0 1 1	1 2 2	3 3 3
1·9	1·378	1·382	1·386	1·389	1·393	1·396	1·400	1·404	1·407	1·411	0 1 1	1 2 2	3 3 3
2·0	1·414	1·418	1·421	1·425	1·428	1·432	1·435	1·439	1·442	1·446	0 1 1	1 2 2	2 3 3
2·1	1·449	1·453	1·456	1·459	1·463	1·466	1·470	1·473	1·476	1·480	0 1 1	1 2 2	2 3 3
2·2	1·483	1·487	1·490	1·493	1·497	1·500	1·503	1·507	1·510	1·513	0 1 1	1 2 2	2 3 3
2·3	1·517	1·520	1·523	1·526	1·530	1·533	1·536	1·539	1·543	1·546	0 1 1	1 2 2	2 3 3
2·4	1·549	1·552	1·556	1·559	1·562	1·565	1·568	1·572	1·575	1·578	0 1 1	1 2 2	2 3 3
2·5	1·581	1·584	1·587	1·591	1·594	1·597	1·600	1·603	1·606	1·609	0 1 1	1 2 2	2 3 3
2·6	1·612	1·616	1·619	1·622	1·625	1·628	1·631	1·634	1·637	1·640	0 1 1	1 2 2	2 2 3
2·7	1·643	1·646	1·649	1·652	1·655	1·658	1·661	1·664	1·667	1·670	0 1 1	1 2 2	2 2 3
2·8	1·673	1·676	1·679	1·682	1·685	1·688	1·691	1·694	1·697	1·700	0 1 1	1 1 2	2 2 3
2·9	1·703	1·706	1·709	1·712	1·715	1·718	1·720	1·723	1·726	1·729	0 1 1	1 1 2	2 2 3
3·0	1·732	1·735	1·738	1·741	1·744	1·746	1·749	1·752	1·755	1·758	0 1 1	1 1 2	2 2 3
3·1	1·761	1·764	1·766	1·769	1·772	1·775	1·778	1·780	1·783	1·786	0 1 1	1 1 2	2 2 3
3·2	1·789	1·792	1·794	1·797	1·800	1·803	1·806	1·808	1·811	1·814	0 1 1	1 1 2	2 2 2
3·3	1·817	1·819	1·822	1·825	1·828	1·830	1·833	1·836	1·838	1·841	0 1 1	1 1 2	2 2 2
3·4	1·844	1·847	1·849	1·852	1·855	1·857	1·860	1·863	1·865	1·868	0 1 1	1 1 2	2 2 2
3·5	1·871	1·873	1·876	1·879	1·881	1·884	1·887	1·889	1·892	1·895	0 1 1	1 1 2	2 2 2
3·6	1·897	1·900	1·903	1·905	1·908	1·910	1·913	1·916	1·918	1·921	0 1 1	1 1 2	2 2 2
3·7	1·924	1·926	1·929	1·931	1·934	1·936	1·939	1·942	1·944	1·947	0 1 1	1 1 2	2 2 2
3·8	1·949	1·952	1·954	1·957	1·960	1·962	1·965	1·967	1·970	1·972	0 1 1	1 1 2	2 2 2
3·9	1·975	1·977	1·980	1·982	1·985	1·987	1·990	1·992	1·995	1·997	0 1 1	1 1 2	2 2 2
4·0	2·000	2·002	2·005	2·007	2·010	2·012	2·015	2·017	2·020	2·022	0 0 1	1 1 1	2 2 2
4·1	2·025	2·027	2·030	2·032	2·035	2·037	2·040	2·042	2·045	2·047	0 0 1	1 1 1	2 2 2
4·2	2·049	2·052	2·054	2·057	2·059	2·062	2·064	2·066	2·069	2·071	0 0 1	1 1 1	2 2 2
4·3	2·074	2·076	2·078	2·081	2·083	2·086	2·088	2·090	2·093	2·095	0 0 1	1 1 1	2 2 2
4·4	2·098	2·100	2·102	2·105	2·107	2·110	2·112	2·114	2·117	2·119	0 0 1	1 1 1	2 2 2
4·5	2·121	2·124	2·126	2·128	2·13	2·133	2·135	2·138	2·140	2·142	0 0 1	1 1 1	2 2 2
4·6	2·145	2·147	2·149	2·152	2·154	2·156	2·159	2·161	2·163	2·166	0 0 1	1 1 1	2 2 2
4·7	2·168	2 170	2·173	2·175	2·177	2·179	2·182	2·184	2·186	2·189	0 0 1	1 1 1	2 2 2
4·8	2·191	2·193	2·195	2·198	2·200	2·202	2·205	2·207	2·209	2·211	0 0 1	1 1 1	2 2 2
4·9	2·214	2·216	2·218	2·220	2·223	2·225	2·227	2·229	2·232	2·234	0 0 1	1 1 1	2 2 2
5·0	2·236	2·238	2·241	2·243	2·245	2·247	2·249	2·252	2·254	2·256	0 0 1	1 1 1	2 2 2
5·1	2·258	2·261	2·263	2·265	2·267	2·269	2·272	2·274	2·276	2·278	0 0 1	1 1 1	2 2 2
5·2	2·280	2·283	2·285	2·287	2·289	2·291	2·293	2·296	2·298	2·300	0 0 1	1 1 1	2 2 2
5·3	2·302	2·304	2·307	2·309	2·311	2·313	2·315	2·317	2·319	2·322	0 0 1	1 1 1	2 2 2
5·4	2·324	2·326	2·328	2 330	2·332	2·335	2·337	2·339	2·341	2·343	0 0 1	1 1 1	1 2 2

	0	1	2	3	4	5	6	7	8	9	Mean Differences.							
											1	2	3	4	5	6	7	8
5·5	2·345	2·347	2·349	2·352	2·354	2·356	2·358	2·360	2·362	2·364	0	0	1	1	1	1	1	2
5·6	2·366	2·369	2·371	2·373	2·375	2·377	2·379	2·381	2·383	2·385	0	0	1	1	1	1	1	2
5·7	2·387	2·390	2·392	2·394	2·396	2·398	2·400	2·402	2·404	2·406	0	0	1	1	1	1	1	2
5·8	2·408	2·410	2·412	2·415	2·417	2·419	2·421	2·423	2·425	2·427	0	0	1	1	1	1	1	2
5·9	2·429	2·431	2·433	2·435	2·437	2·439	2·441	2·443	2·445	2·447	0	0	1	1	1	1	1	2
6·0	2·449	2·452	2·454	2·456	2·458	2·460	2·462	2·464	2·466	2·468	0	0	1	1	1	1	1	2
6·1	2·470	2·472	2·474	2·476	2·478	2·480	2·482	2·484	2·486	2·488	0	0	1	1	1	1	1	2
6·2	2·4900	2·492	2·494	2·496	2·498	2·500	2·502	2·504	2·506	2·508	0	0	1	1	1	1	1	2
6·3	2·510	2·512	2·514	2·516	2·518	2·520	2·522	2·524	2·526	2·528	0	0	1	1	1	1	1	2
6·4	2·530	2·532	2·534	2·536	2·538	2·540	2·542	2·544	2·546	2·548	0	0	1	1	1	1	1	2
6·5	2·550	2·551	2·553	2·555	2·557	2·559	2·561	2·563	2·565	2·567	0	0	1	1	1	1	1	2
6·6	2·569	2·571	2·573	2·575	2·577	2·579	2·581	2·583	2·585	2·587	0	0	1	1	1	1	1	2
6·7	2·588	2·590	2·592	2·594	2·596	2·598	2·600	2·602	2·604	2·606	0	0	1	1	1	1	1	2
6·8	2·608	2·610	2·612	2·613	2·615	2·617	2·619	2·621	2·623	2·625	0	0	1	1	1	1	1	2
6·9	2·627	2·629	2·631	2·632	2·634	2·636	2·638	2·640	2·642	2·644	0	0	1	1	1	1	1	2
7·0	2·646	2·648	2·650	2·651	2·653	2·655	2·657	2·659	2·661	2·663	0	0	1	1	1	1	1	2
7·1	2·665	2·666	2·668	2·670	2·672	2·674	2·676	2·678	2·680	2·681	0	0	1	1	1	1	1	1
7·2	2·683	2·685	2·687	2·689	2·691	2·693	2·694	2·696	2·698	2·700	0	0	1	1	1	1	1	1
7·3	2·702	2·704	2·706	2·707	2·709	2·711	2·713	2·715	2·717	2·718	0	0	1	1	1	1	1	1
7·4	2·720	2·722	2·724	2·726	2·728	2·729	2·731	2·733	2·735	2·737	0	0	1	1	1	1	1	1
7·5	2·739	2·740	2·742	2·744	2·746	2·748	2·750	2·751	2·753	2·755	0	0	1	1	1	1	1	1
7·6	2·757	2·759	2·760	2·762	2·764	2·766	2·768	2·769	2·771	2·773	0	0	1	1	1	1	1	1
7·7	2·775	2·777	2·778	2·780	2·782	2·784	2·786	2·787	2·789	2·791	0	0	1	1	1	1	1	1
7·8	2·793	2·795	2·796	2·798	2·800	2·802	2·804	2·805	2·807	2·809	0	0	1	1	1	1	1	1
7·9	2·811	2·812	2·814	2·816	2·818	2·820	2·821	2·823	2·825	2·827	0	0	1	1	1	1	1	1
8·0	2·828	2·830	2·832	2·834	2·835	2·837	2·839	2·841	2·843	2·844	0	0	1	1	1	1	1	1
8·1	2·846	2·848	2·850	2·851	2·853	2·855	2·857	2·858	2·860	2·862	0	0	1	1	1	1	1	1
8·2	2·864	2·865	2·867	2·869	2·871	2·872	2·874	2·876	2·877	2·879	0	0	1	1	1	1	1	1
8·3	2·881	2·883	2·884	2·886	2·888	2·890	2·891	2·893	2·895	2·897	0	0	1	1	1	1	1	1
8·4	2·898	2·900	2·902	2·903	2·905	2·907	2·909	2·910	2·912	2·914	0	0	1	1	1	1	1	1
8·5	2·915	2·917	2·919	2·921	2·922	2·924	2·926	2·927	2·929	2·931	0	0	1	1	1	1	1	1
8·6	2·933	2·934	2·936	2·938	2·939	2·941	2·943	2·944	2·946	2·948	0	0	1	1	1	1	1	1
8·7	2·950	2·951	2·953	2·955	2·956	2·958	2·960	2·961	2·963	2·965	0	0	1	1	1	1	1	1
8·8	2·966	2·968	2·970	2·972	2·973	2·975	2·977	2·978	2·980	2·982	0	0	1	1	1	1	1	1
8·9	2·983	2·985	2·987	2·988	2·990	2·992	2·993	2·995	2·997	2·998	0	0	1	1	1	1	1	1
9·0	3·000	3·002	3·003	3·005	3·007	3·008	3·010	3·012	3·013	3·015	0	0	0	1	1	1	1	1
9·1	3·017	3·018	3·020	3·022	3·023	3·025	3·027	3·028	3·030	3·032	0	0	0	1	1	1	1	1
9·2	3·033	3·035	3·036	3·038	3·040	3·041	3·043	3·045	3·046	3·048	0	0	0	1	1	1	1	1
9·3	3·050	3·051	3·053	3·055	3·056	3·058	3·059	3·061	3·063	3·064	0	0	0	1	1	1	1	1
9·4	3·066	3·068	3·069	3·071	3·072	3·074	3·076	3·077	3·079	3·081	0	0	0	1	1	1	1	1
9·5	3·082	3·084	3·085	3·087	3·089	3·090	3·092	3·094	3·095	3·097	0	0	0	1	1	1	1	1
9·6	3·098	3·100	3·102	3·103	3·105	3·106	3·108	3·110	3·111	3·113	0	0	0	1	1	1	1	1
9·7	3·114	3·116	3·118	3·119	3·121	3·122	3·124	3·126	3·127	3·129	0	0	0	1	1	1	1	1
9·8	3·130	3·132	3·134	3·135	3·137	3·138	3·140	3·142	3·143	3·145	0	0	0	1	1	1	1	1
9·9	3·146	3·148	3·150	3·151	3·153	3·154	3·156	3·158	3·159	3·161	0	0	0	1	1	1	1	1

SQUARE ROOTS. From 10 to 100

	0	1	2	3	4	5	6	7	8	9	Mean Differences		
											1 2 3	4 5 6	7 8 9
10	3·162	3·178	3·194	3·209	3·225	3·240	3·256	3·271	3·286	3·302	2 3 5	6 8 9	11 12 14
11	3·317	3·332	3·347	3·362	3·376	3·391	3·406	3·421	3·435	3·450	1 3 4	6 7 9	10 12 13
12	3·464	3·479	3·493	3·507	3·521	3·536	3·550	3·564	3·578	3·592	1 3 4	6 7 8	10 11 13
13	3·606	3·619	3·633	3·647	3·661	3·674	3·688	3·701	3·715	3·728	1 3 4	5 7 8	10 11 12
14	3·742	3·755	3·768	3·782	3·795	3·808	3·821	3·834	3·847	3·860	1 3 4	5 7 8	9 11 12
15	3·873	3·886	3·899	3·912	3·924	3·937	3·950	3·962	3·975	3·987	1 3 4	5 6 8	9 10 11
16	4·000	4·012	4·025	4·037	4·050	4·062	4·074	4·087	4·099	4·111	1 2 4	5 6 7	9 10 11
17	4·123	4·135	4·147	4·159	4·171	4·183	4·195	4·207	4·219	4·231	1 2 4	5 6 7	8 10 11
18	4·243	4·254	4·266	4·278	4·290	4·301	4·313	4·324	4·336	4·347	1 2 3	5 6 7	8 9 10
19	4·359	4·370	4·382	4·393	4·405	4·416	4·427	4·438	4·450	4·461	1 2 3	5 6 7	8 9 10
20	4·472	4·483	4·494	4·506	4·517	4·528	4·539	4·550	4·561	4·572	1 2 3	4 6 7	8 9 10
21	4·583	4·593	4·604	4·615	4·626	4·637	4·648	4·658	4·669	4·680	1 2 3	4 5 6	8 9 10
22	4·690	4·701	4·712	4·722	4·733	4·743	4·754	4·764	4·775	4·785	1 2 3	4 5 6	7 8 9
23	4·796	4·806	4·817	4·827	4·837	4·848	4·858	4·868	4·879	4·889	1 2 3	4 5 6	7 8 9
24	4·899	4·909	4·919	4·930	4·940	4·950	4·960	4·970	4·980	4·990	1 2 3	4 5 6	7 8 9
25	5·000	5·010	5·020	5·030	5·040	5·050	5·060	5·070	5·079	5·089	1 2 3	4 5 6	7 8 9
26	5·099	5·109	5·119	5·128	5·138	5·148	5·158	5·167	5·177	5·187	1 2 3	4 5 6	7 8 9
27	5·196	5·206	5·215	5·225	5·235	5·244	5·254	5·263	5·273	5·282	1 2 3	4 5 6	7 8 9
28	5·292	5·301	5·310	5·320	5·329	5·339	5·348	5·357	5·367	5·376	1 2 3	4 5 6	7 7 8
29	5·385	5·394	5·404	5·413	5·422	5·431	5·441	5·450	5·459	5·468	1 2 3	4 5 5	6 7 8
30	5·477	5·486	5·495	5·505	5·514	5·523	5·532	5·541	5·550	5·559	1 2 3	4 4 5	6 7 8
31	5·568	5·577	5·586	5·595	5·604	5·612	5·621	5·630	5·639	5·648	1 2 3	3 4 5	6 7 8
32	5·657	5·666	5·675	5·683	5·692	5·701	5·710	5·718	5·727	5·736	1 2 3	3 4 5	6 7 8
33	5·745	5·753	5·762	5·771	5·779	5·788	5·797	5·805	5·814	5·822	1 2 3	3 4 5	6 7 8
34	5·831	5·840	5·848	5·857	5·865	5·874	5·882	5·891	5·899	5·908	1 2 3	3 4 5	6 7 8
35	5·916	5·925	5·933	5·941	5·950	5·958	5·967	5·975	5·983	5·992	1 2 2	3 4 5	6 7 8
36	6·000	6·008	6·017	6·025	6·033	6·042	6·050	6·058	6·066	6·075	1 2 2	3 4 5	6 7 7
37	6·083	6·091	6·099	6·107	6·116	6·124	6 132	6·140	6·148	6·156	1 2 2	3 4 5	6 7 7
38	6·164	6·173	6·181	6·189	6·197	6·205	6·213	6·221	6·229	6·237	1 2 2	3 4 5	6 6 7
39	6·245	6·253	6·261	6·269	6·277	6·285	6·293	6·301	6·309	6·317	1 2 2	3 4 5	6 6 7
40	6·325	6·332	6·340	6·348	6·356	6·364	6·372	6·380	6·387	6·395	1 2 2	3 4 5	6 6 7
41	6·403	6·411	6·419	6·427	6·434	6·442	6·450	6·458	6·465	6·473	1 2 2	3 4 5	5 6 7
42	6·481	6·488	6·496	6·504	6·512	6·519	6·527	6·535	6·542	6·550	1 2 2	3 4 5	5 6 7
43	6·557	6·565	6·573	6·580	6·588	6·595	6·603	6·611	6·618	6·626	1 2 2	3 4 5	5 6 7
44	6·633	6·641	6·648	6·656	6·663	6·671	6·678	6·686	6·693	6·701	1 2 2	3 4 5	5 6 7
45	6·708	6·716	6·723	6·731	6·738	6·745	6·753	6·760	6·768	6·775	1 1 2	3 4 4	5 6 7
46	6·782	6·790	6·797	6·804	6·812	6·819	6·826	6·834	6·841	6·848	1 1 2	3 4 4	5 6 7
47	6·856	6·863	6·870	6·877	6·885	6·892	6·899	6·907	6·914	6·921	1 1 2	3 4 4	5 6 6
48	6·928	6·935	6·943	6·950	6·957	6·964	6·971	6·979	6·986	6·993	1 1 2	3 4 4	5 6 6
49	7·000	7·007	7·014	7·021	7·029	7·036	7·043	7·050	7·057	7·064	1 1 2	3 4 4	5 6 6
50	7·071	7·078	7·085	7·092	7·099	7·106	7·113	7·120	7·127	7·134	1 1 2	3 4 4	5 6 6
51	7·141	7·148	7·155	7·162	7·169	7·176	7·183	7·190	7·197	7·204	1 1 2	3 4 4	5 6 6
52	7·211	7·218	7·225	7·232	7·239	7·246	7·253	7·259	7·266	7·273	1 1 2	3 3 4	5 6 6
53	7·280	7·287	7·294	7·301	7·308	7·314	7·321	7·328	7·335	7·342	1 1 2	3 3 4	5 5 6
54	7·348	7·355	7·362	7·369	7·376	7·382	7·389	7·396	7·403	7·409	1 1 2	3 3 4	5 5 6

SQUARE ROOTS. From 10 to 100

	0	1	2	3	4	5	6	7	8	9	Mean Differences		
											1 2 3	4 5 6	7 8 9
55	7·416	7·423	7·430	7·436	7·443	7·450	7·457	7·463	7·470	7·477	1 1 2	3 3 4	5 5 6
56	7·483	7·490	7·497	7·503	7·510	7·517	7·523	7·530	7·537	7·543	1 1 2	3 3 4	5 5 6
57	7·550	7·556	7·563	7·570	7·576	7·583	7·589	7·596	7·603	7·609	1 1 2	3 3 4	5 5 6
58	7·616	7·622	7·629	7·635	7·642	7·649	7·655	7·662	7·668	7·675	1 1 2	3 3 4	5 5 6
59	7·681	7·688	7·694	7·701	7·707	7·714	7·720	7·727	7·733	7·740	1 1 2	3 3 4	4 5 6
60	7·746	7·752	7·759	7·765	7·772	7·778	7·785	7·791	7·797	7·804	1 1 2	3 3 4	4 5 6
61	7·810	7·817	7·823	7·829	7·836	7·842	7·849	7·855	7·861	7·868	1 1 2	3 3 4	4 5 6
62	7·874	7·880	7·887	7·893	7·899	7·906	7·912	7·918	7·925	7·931	1 1 2	3 3 4	4 5 6
63	7·937	7·944	7·950	7·956	7·962	7·969	7·975	7·981	7·987	7·994	1 1 2	3 3 4	4 5 6
64	8·000	8·006	8·012	8·019	8·025	8·031	8·037	8·044	8·050	8·056	1 1 2	2 3 4	4 5 6
65	8·062	8·068	8·075	8·081	8·087	8·093	8·099	8·106	8·112	8·118	1 1 2	2 3 4	4 5 6
66	8·124	8·130	8·136	8·142	8·149	8·155	8·161	8·167	8·173	8·179	1 1 2	2 3 4	4 5 5
67	8·185	8·191	8·198	8·204	8·210	8·216	8·222	8·228	8·234	8·240	1 1 2	2 3 4	4 5 5
68	8·246	8·252	8·258	8·264	8·270	8·276	8·283	8·289	8·295	8·301	1 1 2	2 3 4	4 5 5
69	8·307	8·313	8·319	8·325	8·331	8·337	8·343	8·349	8·355	8·361	1 1 2	2 3 4	4 5 5
70	8·367	8·373	8·379	8·385	8·390	8·396	8·402	8·408	8·414	8·420	1 1 2	2 3 4	4 5 5
71	8·426	8·432	8·438	8·444	8·450	8·456	8·462	8·468	8·473	8·479	1 1 2	2 3 4	4 5 5
72	8·485	8·491	8·497	8·503	8·509	8·515	8·521	8·526	8·532	8·538	1 1 2	2 3 3	4 5 5
73	8·544	8·550	8·556	8·562	8·567	8·573	8·579	8·585	8·591	8·597	1 1 2	2 3 3	4 5 5
74	8·602	8·608	8·614	8·620	8·626	8·631	8·637	8·643	8·649	8·654	1 1 2	2 3 3	4 5 5
75	8·660	8·666	8·672	8·678	8·683	8·689	8·695	8·701	8·706	8·712	1 1 2	2 3 3	4 5 5
76	8·718	8·724	8·729	8·735	8·741	8·746	8·752	8·758	8·764	8·769	1 1 2	2 3 3	4 5 5
77	8·775	8·781	8·786	8·792	8·798	8·803	8·809	8·815	8·820	8·826	1 1 2	2 3 3	4 4 5
78	8·832	8·837	8·843	8·849	8·854	8·860	8·866	8·871	8·877	8·883	1 1 2	2 3 3	4 4 5
79	8·888	8·894	8·899	8·905	8·911	8·916	8·922	8·927	8·933	8·939	1 1 2	2 3 3	4 4 5
80	8·944	8·950	8·955	8·961	8·967	8·972	8·978	8·983	8·989	8·994	1 1 2	2 3 3	4 4 5
81	9·000	9·006	9·011	9·017	9·022	9·028	9·033	9·039	9·044	9·050	1 1 2	2 3 3	4 4 5
82	9·055	9·061	9·066	9·072	9·077	9·083	9·088	9·094	9·099	9·105	1 1 2	2 3 3	4 4 5
83	9·110	9·116	9·121	9·127	9·132	9·138	9·143	9·149	9·154	9·160	1 1 2	2 3 3	4 4 5
84	9·165	9·171	9·176	9·182	9·187	9·192	9·198	9·203	9·209	9·214	1 1 2	2 3 3	4 4 5
85	9·220	9·225	9·230	9·236	9·241	9·247	9·252	9·257	9·263	9·268	1 1 2	2 3 3	4 4 5
86	9·274	9·279	9·284	9·290	9·295	9·301	9·305	9·311	9·317	9·322	1 1 2	2 3 3	4 4 5
87	9·327	9·333	9·338	9·343	9·349	9·354	9·359	9·365	9·370	9·375	1 1 2	2 3 3	4 4 5
88	9·381	9·386	9·391	9·397	9·402	9·407	9·413	9·418	9·423	9·429	1 1 2	2 3 3	4 4 5
89	9·434	9·439	9·445	9·450	9·455	9·460	9·466	9·471	9·476	9·482	1 1 2	2 3 3	4 4 5
90	9·487	9·492	9·497	9·503	9·508	9·513	9·518	9·524	9·529	9·534	1 1 2	2 3 3	4 4 5
91	9·539	9·545	9·550	9·555	9·560	9·566	9·571	9·576	9·581	9·586	1 1 2	2 3 3	4 4 5
92	9·592	9·597	9·602	9·607	9·612	9·618	9·623	9·628	9·633	9·638	1 1 2	2 3 3	4 4 5
93	9·644	9·649	9·654	9·659	9·664	9·670	9·675	9·680	9·685	9·690	1 1 2	2 3 3	4 4 5
94	9·695	9·701	9·706	9·711	9·716	9·721	9·726	9·731	9·737	9·742	1 1 2	2 3 3	4 4 5
95	9·747	9·752	9·757	9·762	9·767	9·772	9·778	9·783	9·788	9·793	1 1 2	2 3 3	4 4 5
96	9·798	9·803	9·808	9·813	9·818	9·823	9·829	9·834	9·839	9·844	1 1 2	2 3 3	4 4 5
97	9·849	9·854	9·859	9·864	9·869	9·874	9·879	9·884	9·889	9·894	1 1 1	2 3 3	4 4 5
98	9·899	9·905	9·910	9·915	9·920	9·925	9·930	9·935	9·940	9·945	0 1 1	2 2 3	3 4 4
99	9·950	9·955	9·960	9·965	9·970	9·975	9·980	9·985	9·990	9·995	0 1 1	2 2 3	3 4 4

RECIPROCALS OF NUMBERS. FROM 1 TO 10

[*Numbers in difference columns to be subtracted, not added.*]

	0	1	2	3	4	5	6	7	8	9	Mean Differences		
											1 2 3	4 5 6	7 8 9
1·0	1·000	9901	9804	9709	9615	9524	9434	9346	9259	9174			
1·1	·9091	9009	8929	8850	8772	8696	8621	8547	8475	8403			
1·2	·8333	8264	8197	8130	8065	8000	7937	7874	7813	7752			
1·3	·7692	7634	7576	7519	7463	7407	7353	7299	7246	7194			
1·4	·7143	7092	7042	6993	6944	6897	6849	6803	6757	6711	5 10 14	19 24 29	33 38 43
1·5	·6667	6623	6579	6536	6494	6452	6410	6369	6329	6289	4 8 13	17 21 25	29 33 38
1·6	·6250	6211	6173	6135	6098	6061	6024	5988	5952	5917	4 7 11	15 18 22	26 29 33
1·7	·5882	5848	5814	5780	5747	5714	5682	5650	5618	5587	3 6 10	13 16 20	23 26 29
1·8	·5556	5525	5495	5464	5435	5405	5376	5348	5319	5291	3 6 9	12 15 17	20 23 26
1·9	·5263	5236	5208	5181	5155	5128	5102	5076	5051	5025	3 5 8	11 13 16	18 21 24
2·0	·5000	4975	4950	4926	4902	4878	4854	4831	4808	4785	2 5 7	10 12 14	17 19 21
2·1	·4762	4739	4717	4695	4673	4651	4630	4608	4587	4566	2 4 7	9 11 13	15 17 20
2·2	·4545	4525	4505	4484	4464	4444	4425	4405	4386	4367	2 4 6	8 10 12	14 16 18
2·3	·4348	4329	4310	4292	4274	4255	4237	4219	4202	4184	2 4 5	7 9 11	13 14 16
2·4	·4167	4149	4132	4115	4098	4082	4065	4049	4032	4016	2 3 5	7 8 10	12 13 15
2·5	·4000	3984	3968	3953	3937	3922	3906	3891	3876	3861	2 3 5	6 8 9	11 12 14
2·6	·3846	3831	3817	3802	3788	3774	3759	3745	3731	3717	1 3 4	6 7 8	10 11 13
2·7	·3704	3690	3676	3663	3650	3636	3623	3610	3597	3584	1 3 4	5 7 8	9 11 12
2·8	·3571	3559	3546	3534	3521	3509	3497	3484	3472	3460	1 2 4	5 6 7	9 10 11
2·9	·3448	3436	3425	3413	3401	3390	3378	3367	3356	3344	1 2 3	5 6 7	8 9 10
3·0	·3333	3322	3311	3300	3289	3279	3268	3257	3247	3236	1 2 3	4 5 6	7 9 10
3·1	·3226	3215	3205	3195	3185	3175	3165	3155	3145	3135	1 2 3	4 5 6	7 8 9
3·2	·3125	3115	3106	3096	3086	3077	3067	3058	3049	3040	1 2 3	4 5 6	7 8 9
3·3	·3030	3021	3012	3003	2994	2985	2976	2967	2959	2950	1 2 3	4 4 5	6 7 8
3·4	·2941	2933	2924	2915	2907	2899	2890	2882	2874	2865	1 2 3	3 4 5	6 7 8
3·5	·2857	2849	2841	2833	2825	2817	2809	2801	2793	2786	1 2 2	3 4 5	6 6 7
3·6	·2778	2770	2762	2755	2747	2740	2732	2725	2717	2710	1 2 2	3 4 5	5 6 7
3·7	·2703	2695	2688	2681	2674	2667	2660	2653	2646	2639	1 1 2	3 4 4	5 6 6
3·8	·2632	2625	2618	2611	2604	2597	2591	2584	2577	2571	1 1 2	3 3 4	5 5 6
3·9	·2564	2558	2551	2545	2538	2532	2525	2519	2513	2506	1 1 2	3 3 4	4 5 6
4·0	·2500	2494	2488	2481	2475	2469	2463	2457	2451	2445	1 1 2	2 3 4	4 5 5
4·1	·2439	2433	2427	2421	2415	2410	2404	2398	2392	2387	1 1 2	2 3 3	4 5 5
4·2	·2381	2375	2370	2364	2358	2353	2347	2342	2336	2331	1 1 2	2 3 3	4 4 5
4·3	·2326	2320	2315	2309	2304	2299	2294	2288	2283	2278	1 1 2	2 3 3	4 4 5
4·4	·2273	2268	2262	2257	2252	2247	2242	2237	2232	2227	1 1 2	2 3 3	4 4 5
4·5	·2222	2217	2212	2208	2203	2198	2193	2188	2183	2179	0 1 1	2 2 3	3 4 4
4·6	·2174	2169	2165	2160	2155	2151	2146	2141	2137	2132	0 1 1	2 2 3	3 4 4
4·7	·2128	2123	2119	2114	2110	2105	2101	2096	2092	2088	0 1 1	2 2 3	3 4 4
4·8	·2083	2079	2075	2070	2066	2062	2058	2053	2049	2045	0 1 1	2 2 3	3 3 4
4·9	·2041	2037	2033	2028	2024	2020	2016	2012	2008	2004	0 1 1	2 2 2	3 3 4
5·0	·2000	1996	1992	1988	1984	1980	1976	1972	1969	1965	0 1 1	2 2 2	3 3 4
5·1	·1961	1957	1953	1949	1946	1942	1938	1934	1931	1927	0 1 1	2 2 2	3 3 3
5·2	·1923	1919	1916	1912	1908	1905	1901	1898	1894	1890	0 1 1	1 2 2	3 3 3
5·3	·1887	1883	1880	1876	1873	1869	1866	1862	1859	1855	0 1 1	1 2 2	2 3 3
5·4	·1852	1848	1845	1842	1838	1835	1832	1828	1825	1821	0 1 1	1 2 2	2 3 3

RECIPROCALS OF NUMBERS. FROM 1 TO 10

[Numbers in difference columns to be subtracted, not added.]

	0	1	2	3	4	5	6	7	8	9	Mean Differences		
											1 2 3	4 5 6	7 8 9
5·5	·1818	1815	1812	1808	1805	1802	1799	1795	1792	1789	0 1 1	1 2 2	2 3 3
5·6	·1786	1783	1779	1776	1773	1770	1767	1764	1761	1757	0 1 1	1 2 2	2 3 3
5·7	·1754	1751	1748	1745	1742	1739	1736	1733	1730	1727	0 1 1	1 1 2	2 2 3
5·8	·1724	1721	1718	1715	1712	1709	1706	1704	1701	1698	0 1 1	1 1 2	2 2 3
5·9	·1695	1692	1689	1686	1684	1681	1678	1675	1672	1669	0 1 1	1 1 2	2 2 3
6·0	·1667	1664	1661	1658	1656	1653	1650	1647	1645	1642	0 1 1	1 1 2	2 2 3
6·1	·1639	1637	1634	1631	1629	1626	1623	1621	1618	1616	0 1 1	1 1 2	2 2 2
6·2	·1613	1610	1608	1605	1603	1600	1597	1595	1592	1590	0 1 1	1 1 2	2 2 2
6·3	·1587	1585	1582	1580	1577	1575	1572	1570	1567	1565	0 0 1	1 1 1	2 2 2
6·4	·1562	1560	1558	1555	1553	1550	1548	1546	1543	1541	0 0 1	1 1 1	2 2 2
6·5	·1538	1536	1534	1531	1529	1527	1524	1522	1520	1517	0 0 1	1 1 1	2 2 2
6·6	·1515	1513	1511	1508	1506	1504	1502	1499	1497	1495	0 0 1	1 1 1	2 2 2
6·7	·1493	1490	1488	1486	1484	1481	1479	1477	1475	1473	0 0 1	1 1 1	2 2 2
6·8	·1471	1468	1466	1464	1462	1460	1458	1456	1453	1451	0 0 1	1 1 1	2 2 2
6·9	·1449	1447	1445	1443	1441	1439	1437	1435	1433	1431	0 0 1	1 1 1	2 2 2
7·0	·1429	1427	1425	1422	1420	1418	1416	1414	1412	1410	0 0 1	1 1 1	1 2 2
7·1	·1408	1406	1404	1403	1401	1399	1397	1395	1393	1391	0 0 1	1 1 1	1 2 2
7·2	·1389	1387	1385	1383	1381	1379	1377	1376	1374	1372	0 0 1	1 1 1	1 2 2
7·3	·1370	1368	1366	1364	1362	1361	1359	1357	1355	1353	0 0 1	1 1 1	1 2 2
7·4	·1351	1350	1348	1346	1344	1342	1340	1339	1337	1335	0 0 1	1 1 1	1 1 2
7·5	·1333	1332	1330	1328	1326	1325	1323	1321	1319	1318	0 0 1	1 1 1	1 1 2
7·6	·1316	1314	1312	1311	1309	1307	1305	1304	1302	1300	0 0 1	1 1 1	1 1 2
7·7	·1299	1297	1295	1294	1292	1290	1289	1287	1285	1284	0 0 0	1 1 1	1 1 1
7·8	·1282	1280	1279	1277	1276	1274	1272	1271	1269	1267	0 0 0	1 1 1	1 1 1
7·9	·1266	1264	1263	1261	1259	1258	1256	1255	1253	1252	0 0 0	1 1 1	1 1 1
8·0	·1250	1248	1247	1245	1244	1242	1241	1239	1238	1236	0 0 0	1 1 1	1 1 1
8·1	·1235	1233	1232	1230	1229	1227	1225	1224	1222	1221	0 0 0	1 1 1	1 1 1
8·2	·1220	1218	1217	1215	1214	1212	1211	1209	1208	1206	0 0 0	1 1 1	1 1 1
8·3	·1205	1203	1202	1200	1199	1198	1196	1195	1193	1192	0 0 0	1 1 1	1 1 1
8·4	·1190	1189	1188	1186	1185	1183	1182	1181	1179	1178	0 0 0	1 1 1	1 1 1
8·5	·1176	1175	1174	1172	1171	1170	1168	1167	1166	1164	0 0 0	1 1 1	1 1 1
8·6	·1163	1161	1160	1159	1157	1156	1155	1153	1152	1151	0 0 0	1 1 1	1 1 1
8·7	·1149	1148	1147	1145	1144	1143	1142	1140	1139	1138	0 0 0	1 1 1	1 1 1
8·8	·1136	1135	1134	1133	1131	1130	1129	1127	1126	1125	0 0 0	1 1 1	1 1 1
8·9	·1124	1122	1121	1120	1119	1117	1116	1115	1114	1112	0 0 0	1 1 1	1 1 1
9·0	·1111	1110	1109	1107	1106	1105	1104	1103	1101	1100	0 0 0	1 1 1	1 1 1
9·1	·1099	1098	1096	1095	1094	1093	1092	1090	1089	1088	0 0 0	0 1 1	1 1 1
9·2	·1087	1086	1085	1083	1082	1081	1080	1079	1078	1076	0 0 0	0 1 1	1 1 1
9·3	·1075	1074	1073	1072	1071	1070	1068	1067	1066	1065	0 0 0	0 1 1	1 1 1
9·4	·1064	1063	1062	1060	1059	1058	1057	1056	1055	1054	0 0 0	0 1 1	1 1 1
9·5	·1053	1052	1050	1049	1048	1047	1046	1045	1044	1043	0 0 0	0 1 1	1 1 1
9·6	·1042	1041	1039	1038	1037	1036	1035	1034	1033	1032	0 0 0	0 1 1	1 1 1
9·7	·1031	1030	1029	1028	1027	1026	1025	1024	1022	1021	0 0 0	0 1 1	1 1 1
9·8	·1020	1019	1018	1017	1016	1015	1014	1013	1012	1011	0 0 0	0 1 1	1 1 1
9·9	·1010	1009	1008	1007	1006	1005	1004	1003	1002	1001	0 0 0	0 0 1	1 1 1

EXPONENTIAL AND HYPERBOLIC FUNCTIONS

x	e^x	e^{-x}	sinh x	cosh x	x	e^x	e^{-x}	sinh x	cosh x
·02	1·0202	·9802	·0200	1·0002	1·0	2·7183	·3679	1·1752	1·5431
·04	1·0408	·9608	·0400	1·0008	1·1	3·0042	·3329	1·3356	1·6685
·06	1·0618	·9418	·0600	1·0018	1·2	3·3201	·3012	1·5095	1·8107
·08	1·0833	·9231	·0801	1·0032	1·3	3·6693	·2725	1·6984	1·9709
·10	1·1052	·9048	·1002	1·0050	1·4	4·0552	·2466	1·9043	2·1509
·11	1·1163	·8958	·1102	1·0061	1·5	4·4817	·2231	2·1293	2·3524
·12	1·1275	·8869	·1203	1·0072	1·6	4·9530	·2019	2·3756	2·5775
·13	1·1388	·8781	·1304	1·0085	1·7	5·4739	·1827	2·6456	2·8283
·14	1·1503	·8694	·1405	1·0098	1·8	6·0497	·1653	2·9422	3·1075
·15	1·1618	·8607	·1506	1·0113	1·9	6·6859	·1496	3·2682	3·4177
·16	1·1735	·8521	·1607	1·0128	2·0	7·3891	·1353	3·6269	3·7622
·17	1·1853	·8437	·1708	1·0145	2·1	8·1662	·1225	4·0219	4·1443
·18	1·1972	·8353	·1810	1·0162	2·2	9·0250	·1108	4·4571	4·5679
·19	1·2092	·8270	·1911	1·0181	2·3	9·9742	·1003	4·9370	5·0372
·20	1·2214	·8187	·2013	1·0201	2·4	11·023	·0907	5·4662	5·5569
·21	1·2337	·8106	·2115	1·0221	2·5	12·182	·0821	6·0502	6·1323
·22	1·2461	·8025	·2218	1·0243	2·6	13·464	·0743	6·6947	6·7690
·23	1·2586	·7945	·2320	1·0266	2·7	14·880	·0672	7·4063	7·4735
·24	1·2712	·7866	·2423	1·0289	2·8	16·445	·0608	8·1919	8·2527
·25	1·2840	·7788	·2526	1·0314	2·9	18·174	·0550	9·0596	9·1146
·26	1·2969	·7711	·2629	1·0340	3·0	20·085	·0498	10·018	10·068
·27	1·3100	·7634	·2733	1·0367	3·1	22·198	·0450	11·076	11·121
·28	1·3231	·7558	·2837	1·0395	3·2	24·532	·0408	12·246	12·287
·29	1·3364	·7483	·2941	1·0423	3·3	27·113	·0369	13·538	13·575
·30	1·3499	·7408	·3045	1·0453	3·4	29·964	·0334	14·965	14·999
·31	1·3634	·7335	·3150	1·0484	3·5	33·115	·0302	16·543	16·573
·32	1·3771	·7261	·3255	1·0516	3·6	36·598	·0273	18·285	18·313
·33	1·3910	·7189	·3360	1·0550	3·7	40·447	·0247	20·211	20·236
·34	1·4050	·7118	·3466	1·0584	3·8	44·701	·0224	22·339	22·362
·35	1·4191	·7047	·3572	1·0619	3·9	49·402	·0202	24·691	24·711
·36	1·4333	·6977	·3678	1·0655	4·0	54·598	·0183	27·290	27·308
·37	1·4477	·6907	·3785	1·0692	4·1	60·340	·0166	30·162	30·178
·38	1·4623	·6839	·3892	1·0731	4·2	66·686	·0150	33·336	33·351
·39	1·4770	·6771	·4000	1·0770	4·3	73·700	·0136	36·843	36·857
·40	1·4918	·6703	·4107	1·0811	4·4	81·451	·0123	40·719	40·732
·41	1·5068	·6636	·4216	1·0852	4·5	90·017	·0111	45·003	45·014
·42	1·5220	·6570	·4325	1·0895	4·6	99·484	·0100	49·737	49·747
·43	1·5373	·6505	·4434	1·0939	4·7	109·95	·00910	54·969	54·978
·44	1·5527	·6440	·4543	1·0984	4·8	121·51	·00823	60·751	60·759
·45	1·5683	·6376	·4653	1·1030	4·9	134·29	·00745	67·141	67·149
·46	1·5841	·6313	·4764	1·1077	5·0	148·41	·00674	74·203	74·210
·47	1·6000	·6250	·4875	1·1125	5·1	164·02	·00610	82·008	82·014
·48	1·6161	·6188	·4986	1·1174	5·2	181·27	·00552	90·633	90·639
·49	1·6323	·6126	·5098	1·1225	5·3	200·34	·00499	100·17	100·17
·50	1·6487	·6065	·5211	1·1276	5·4	221·41	·00452	110·70	110·71
·6	1·8221	·5488	·6367	1·1855	5·5	244·69	·00409	122·34	122·35
·7	2·0138	·4966	·7586	1·2552	5·6	270·43	·00370	135·21	135·21
·8	2·2255	·4493	·8881	1·3374	5·7	298·87	·00335	149·43	149·43
·9	2·4596	·4066	1·0265	1·4331	5·8	330·30	·00303	165·15	165·15
					5·9	365·04	·00274	182·52	182·52
					6·0	403·43	·00248	201·71	201·72

$\cosh x = \frac{1}{2}(e^x + e^{-x})$, $\sinh x = \frac{1}{2}(e^x - e^{-x})$.

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